

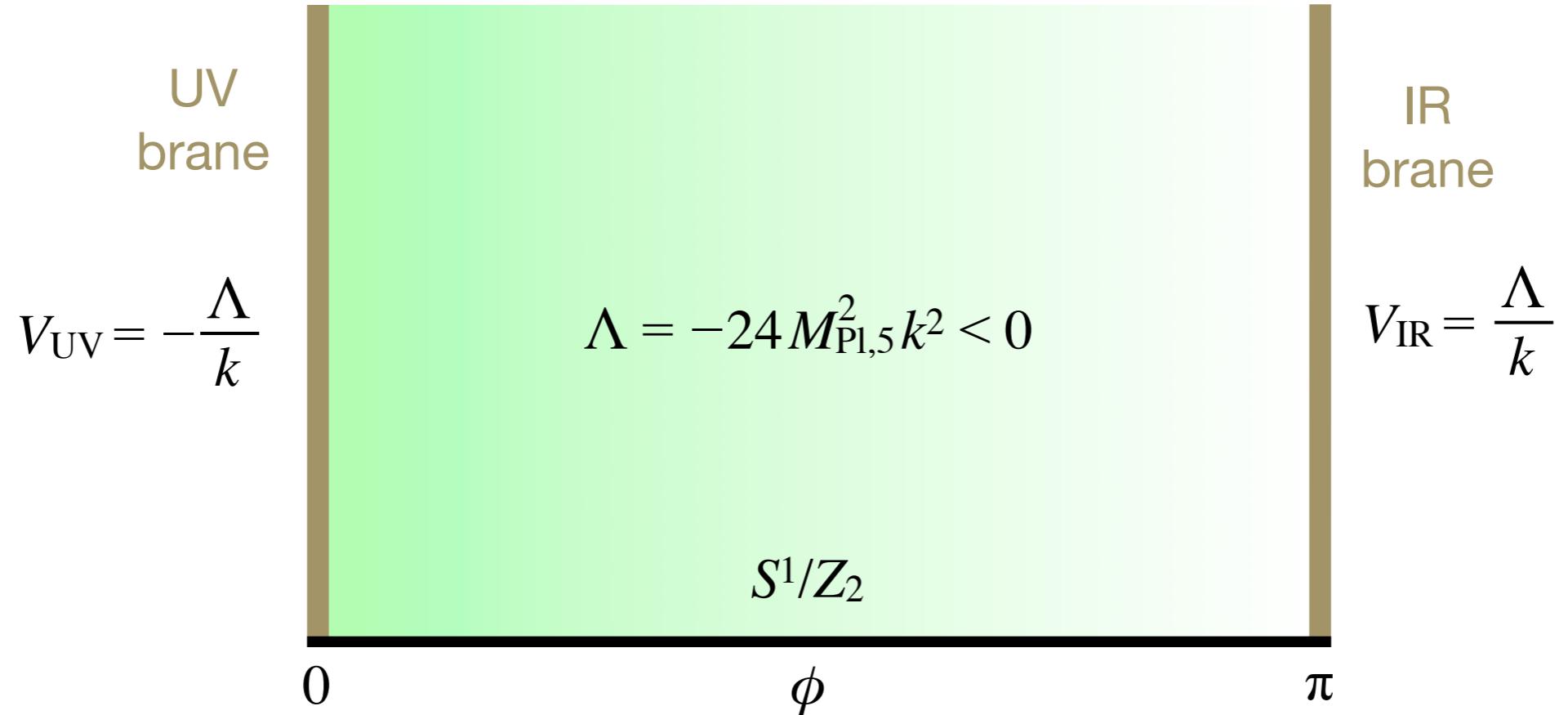


# Quark flavor in RS: Overtime

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with M. Bauer, S. Casagrande, F. Goertz,  
L. Gründer, M. Neubert, and T. Pfoh,  
arXiv:0807.4537 and papers in preparation

# RS model: Geometry



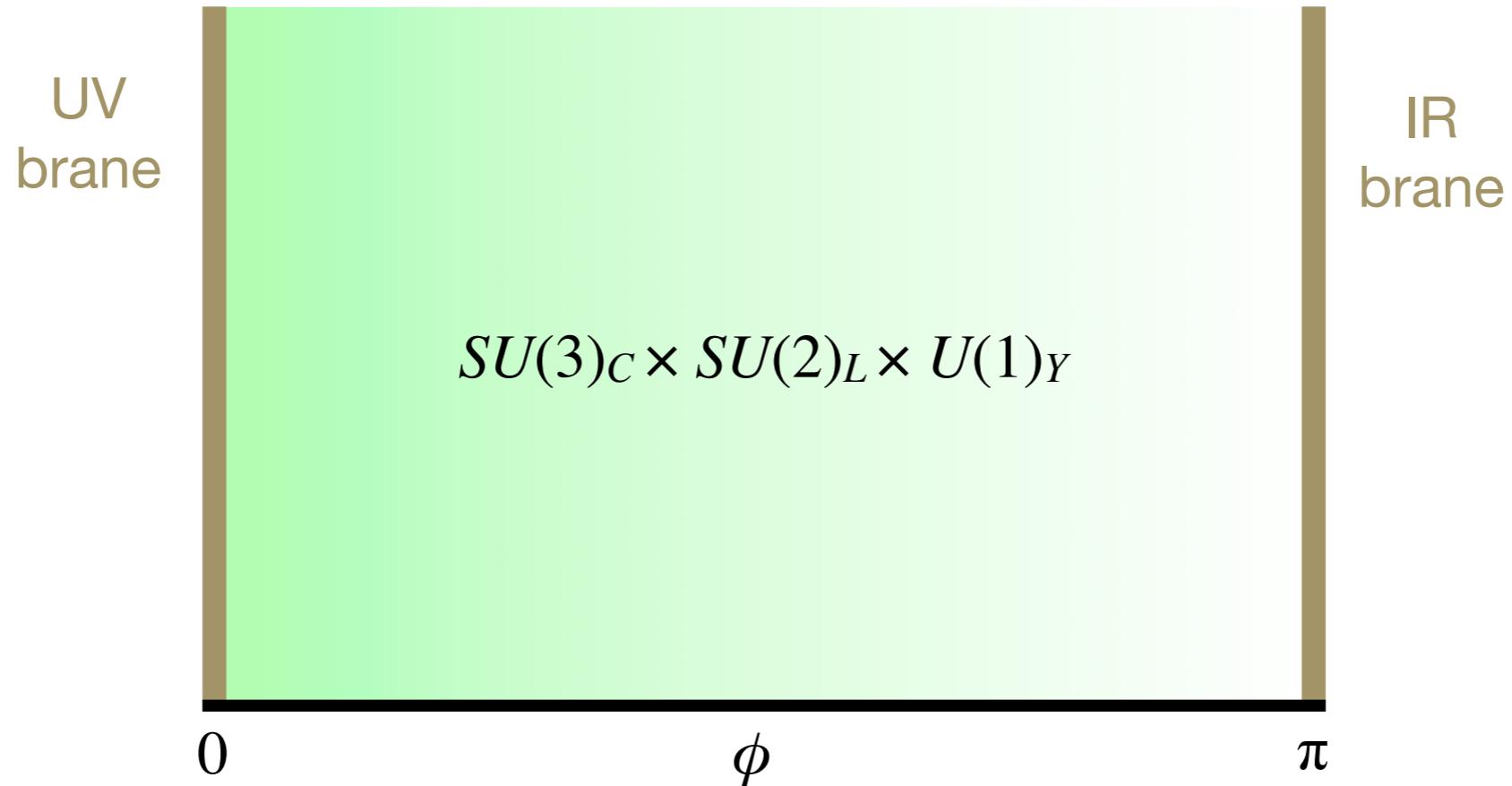
Slice of  $\text{AdS}_5$  with curvature  $k$ :

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu - r^2 d\phi^2, \quad \sigma = kr|\phi|$$

$$\epsilon = \frac{M_W}{M_{\text{Pl}}} = e^{-kr\pi} \approx 10^{-16}, \quad L = -\ln \epsilon \approx 37, \quad M_{\text{KK}} = k\epsilon = \text{few TeV}$$

# RS model: Gauge sector

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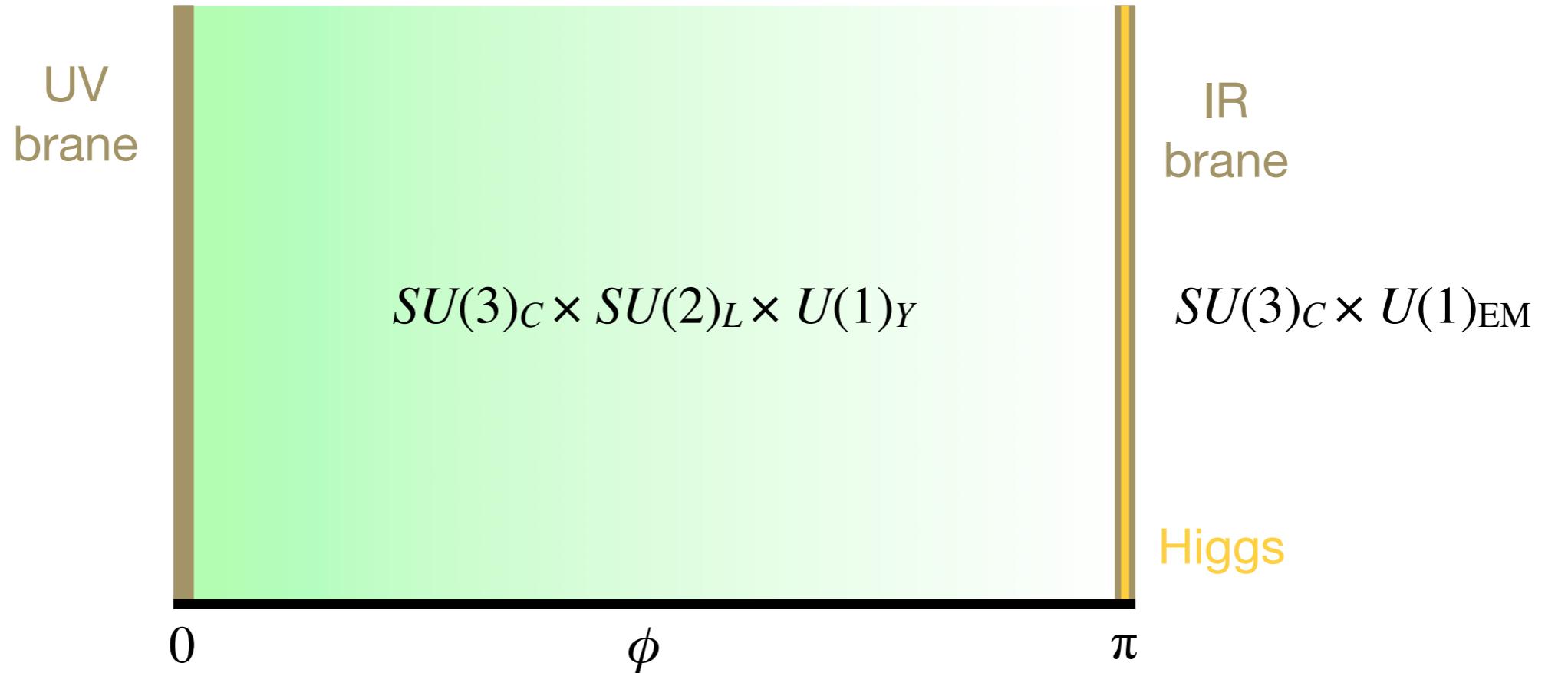


Gauge theory in bulk:

$$\mathcal{L}_{\text{gauge}} = \frac{\sqrt{G}}{r} G^{KM} G^{LN} \left( -\frac{1}{4} F_{KL}^A F_{MN}^A - \frac{1}{4} W_{KL}^a W_{MN}^a - \frac{1}{4} B_{KL} B_{MN} \right)$$

5D fields are decomposed into 4D KK states with suitable parity assignments

# RS model: Higgs sector

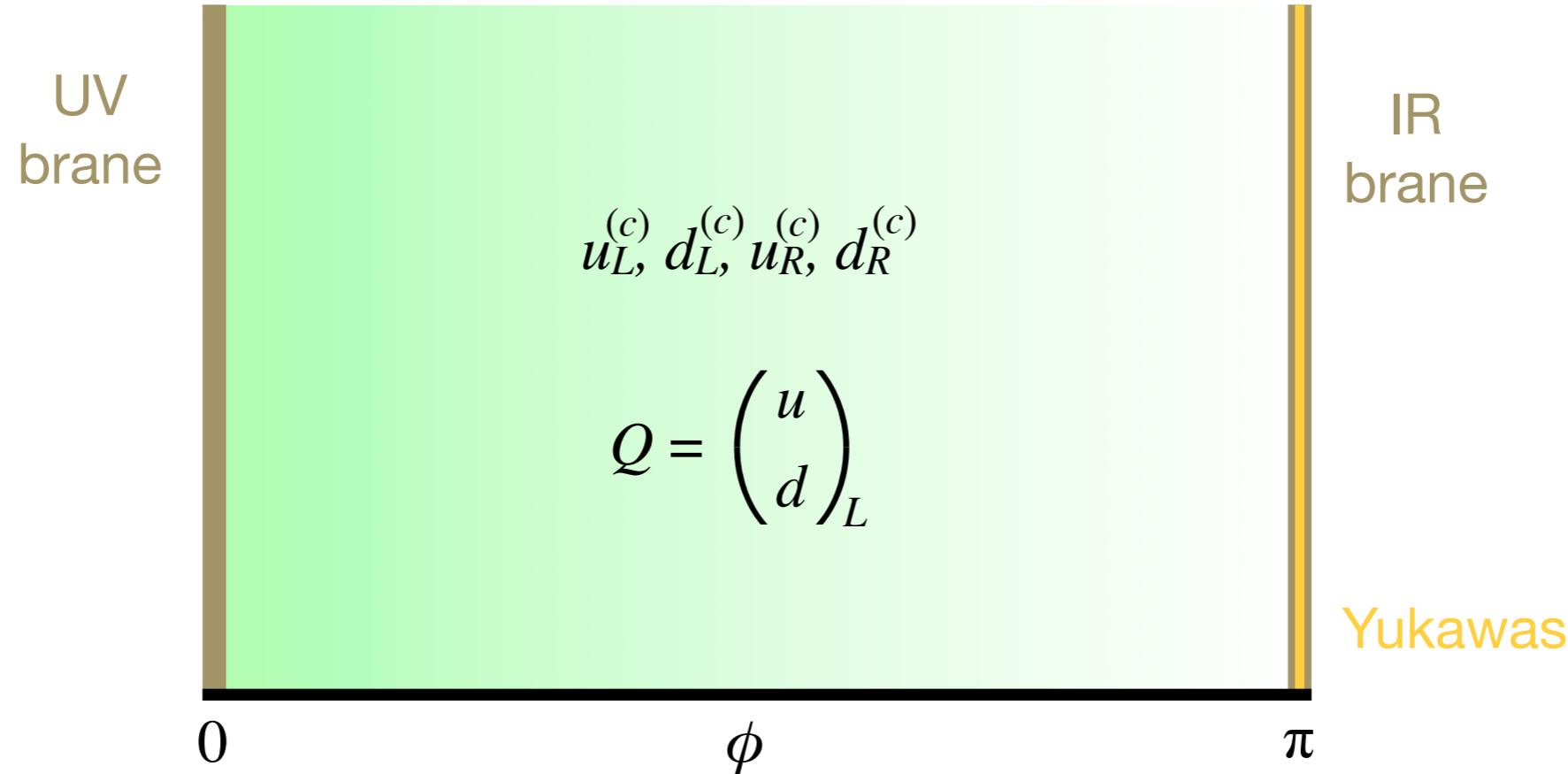


Brane-localized Higgs sector:

$$\mathcal{L}_{\text{Higgs}} = \frac{\delta(|\phi| - \pi)}{r} \left[ (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \right], \quad V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

After EWSB gauge bosons and KK modes get masses  $m_0, m_1 \approx 2.45 M_{\text{KK}}, \dots$

# RS model: Fermion sector

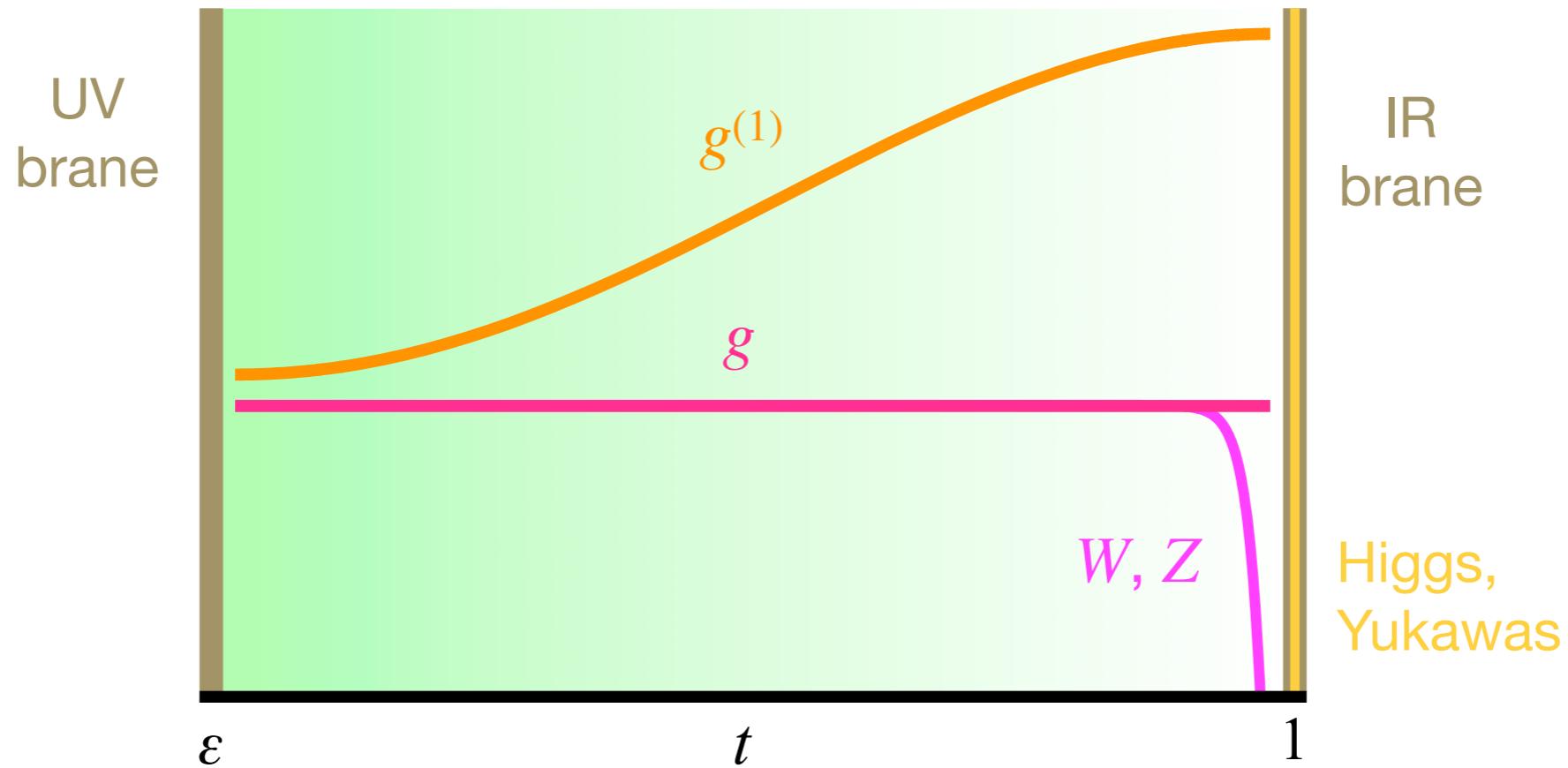


Bulk fermions and brane-localized Yukawas:

$$\mathcal{L}_{\text{ferm}} = e^{-3\sigma} \text{sgn}(\phi) (\bar{Q} M_Q Q + \bar{q}^c M_q q^c) + \frac{\sqrt{2} v e^{-5\sigma}}{kr} \delta(|\phi| - \pi) [\bar{q}_L Y_q q_R^c + \text{h.c.}] + \dots$$

Parameters  $c_{Q,q} = \pm M_{Q,q}/k$  control localization of fermion profiles in 5<sup>th</sup> dimension

# RS model: Gauge boson profiles\*

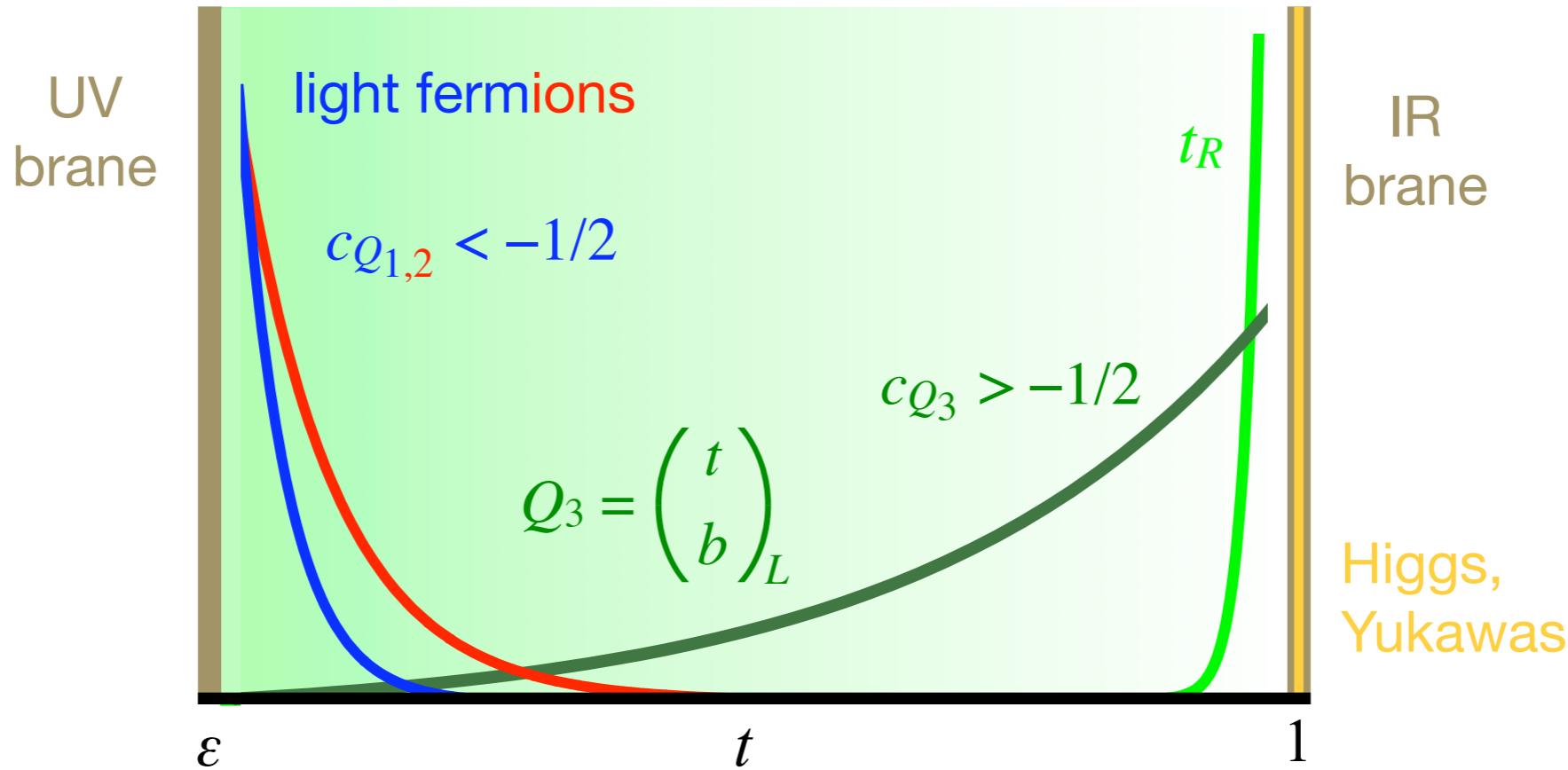


Profiles of gauge fields:

$$\chi_{g,\gamma}(\phi) = \frac{1}{\sqrt{2\pi}}, \quad \chi_{W,Z}(\phi) \approx \frac{1}{\sqrt{2\pi}} \left[ 1 + \frac{m_{W,Z}^2}{M_{\text{KK}}^2} \left( 1 - \frac{1}{L} + t^2 (1 - 2L - 2 \ln t) \right) \right]$$

Wave functions of heavy gauge bosons and KK excitations peaked at IR brane

# RS model: Fermion profiles\*

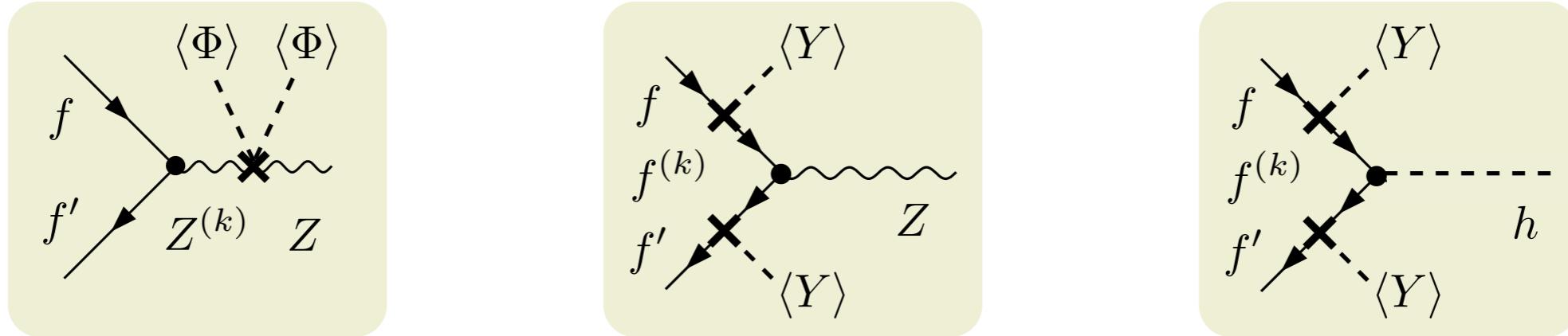


Profiles of fermion fields:

$$C_n^{(A)}(\phi) \approx \sqrt{\frac{L\epsilon}{\pi}} F_{c_A} t^{c_A}, \quad S_n^{(A)}(\phi) \approx \pm \text{sgn}(\phi) \sqrt{\frac{L\epsilon}{\pi}} \frac{m_n}{M_{\text{KK}}} \left( \frac{t^{-c_A}}{F_{c_A}} + \frac{t^{1+c_A} - t^{-c_A}}{1 - 2c_A} F_{c_A} \right)$$

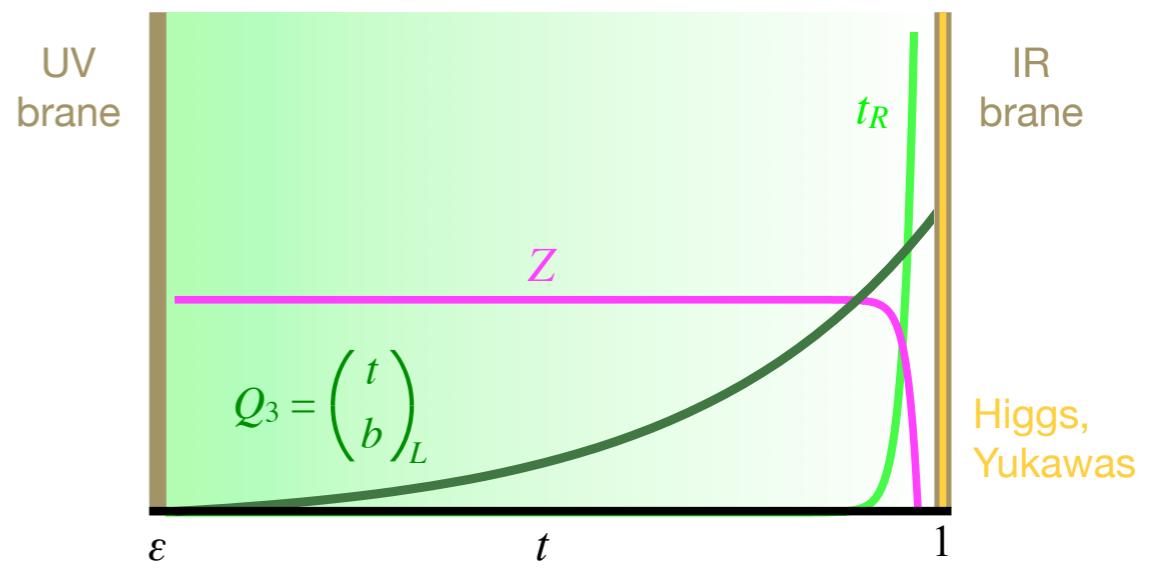
Top quark lives in IR to generate its large mass, while light fermions live in UV

# Sources of flavor violation: Light weak bosons

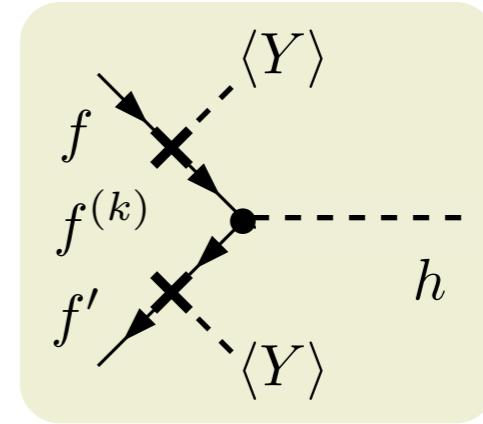
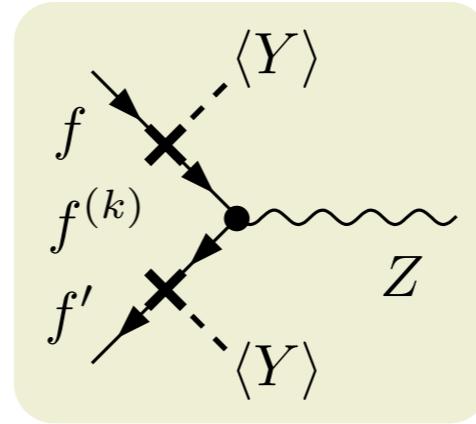
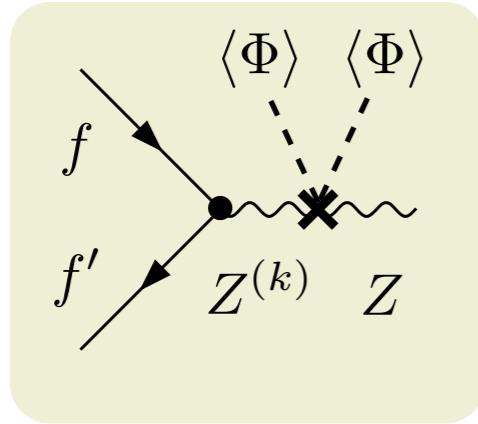


## Couplings of light weak bosons:

- ▶ flavor violation from modification of  $W, Z$  boson profiles due to EWSB on IR brane\*
- ▶ flavor violation from non-orthonormality of fermion profiles interpreted as mixing of  $SU(2)_L$  singlet and doublets



# Sources of flavor violation: Light weak bosons



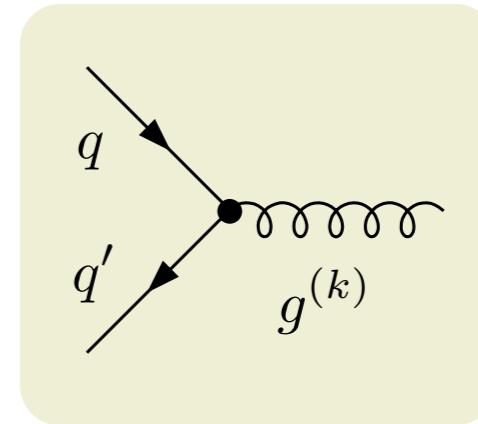
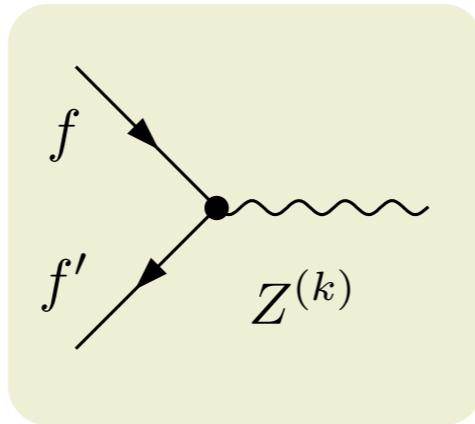
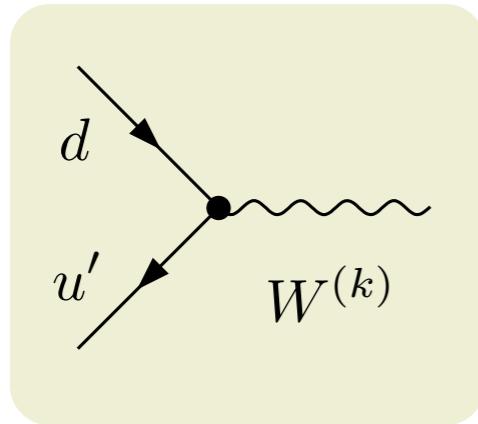
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$$\int_{-\pi}^{\pi} d\phi e^\sigma C_m^{(A)} C_n^{(A)} = \delta_{mn} + \Delta C_{mn}^{(A)},$$

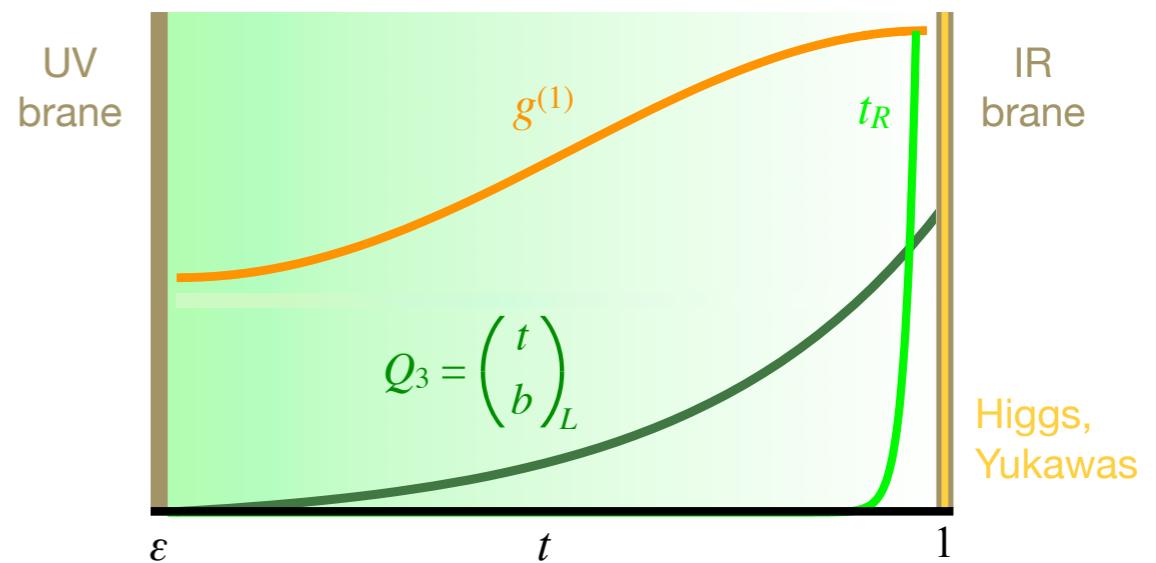
$$\int_{-\pi}^{\pi} d\phi e^\sigma S_m^{(A)} S_n^{(A)} = \delta_{mn} + \Delta S_{mn}^{(A)}$$

# Sources of flavor violation: KK gauge bosons



## Couplings of KK gauge bosons\*:

- ▶ flavor violation from non-trivial overlap integrals of KK gauge-boson profiles with SM fermion wave functions
- ▶ dominant corrections arise typically from vertices involving KK gluons



\*Burdman, hep-ph/0310144; Agashe *et al.*, hep-ph/0406101, hep-ph/0408134

# Mixing matrices: Gauge and KK boson effects

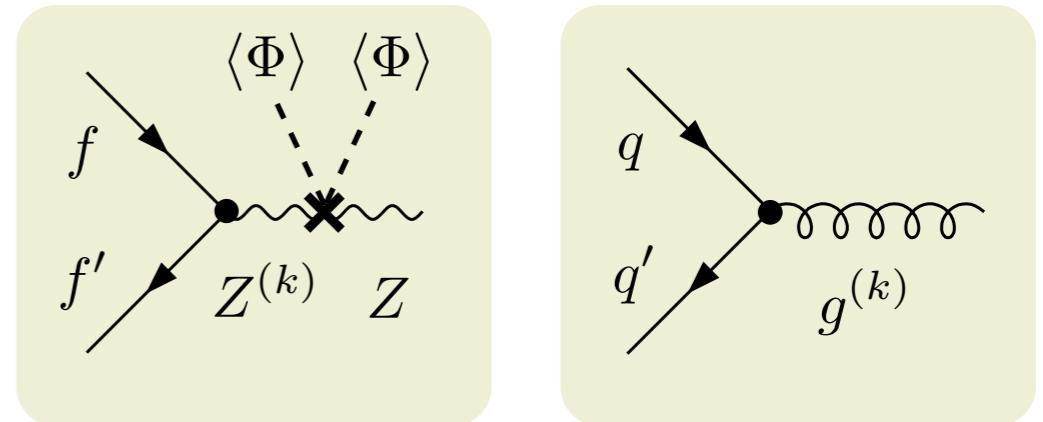
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$$(\Delta_Q)_{ij} \rightarrow \left( \mathbf{U}_q^\dagger \operatorname{diag} \left[ \frac{F_{c_{Q_i}}^2}{3 + 2c_{Q_i}} \right] \mathbf{U}_q \right)_{ij}, \quad (\Delta_q)_{ij}, (\Delta'_q)_{ij}: Q_i \rightarrow q_i, \mathbf{U}_q \rightarrow \mathbf{W}_q,$$

$$(\Delta'_Q)_{ij} \rightarrow \left( \mathbf{U}_q^\dagger \operatorname{diag} \left[ \frac{5 + 2c_{Q_i}}{2(3 + 2c_{Q_i})^2} F_{c_{Q_i}}^2 \right] \mathbf{U}_q \right)_{ij}, \quad \mathbf{V}_{\text{CKM}} \rightarrow \mathbf{U}_u^\dagger \mathbf{U}_d$$

Effects due to gauge-boson profiles\*:

- ▶ in flavor eigenbasis couplings of gauge bosons and KK modes to fermions are flavor-diagonal but non-universal
- ▶ after transformation to mass eigenbasis via left- and right-handed rotations  $\mathbf{U}_q$  and  $\mathbf{W}_q$ , tree-level FCNCs arise



# Mixing matrices: Fermion mixing

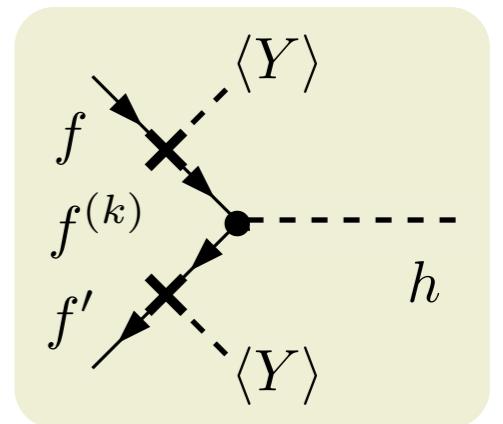
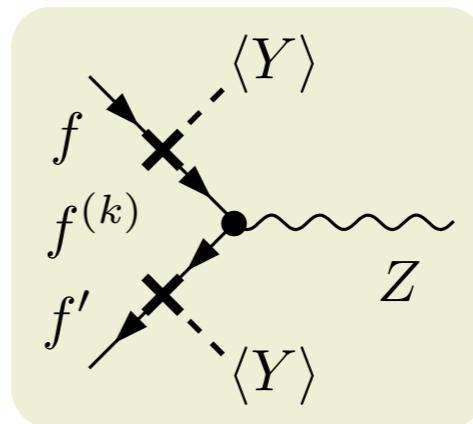
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$$(\delta_Q)_{ij} \rightarrow \left( \mathbf{x}_q \, \mathbf{W}_q^\dagger \, \text{diag} \left[ \frac{1}{1 - 2c_{q_i}} \left( \frac{1}{F_{c_{q_i}}^2} - 1 + \frac{F_{c_{q_i}}^2}{3 + 2c_{q_i}} \right) \right] \mathbf{W}_q \, \mathbf{x}_q \right)_{ij},$$

$$(\delta_q)_{ij}: c_{q_i} \rightarrow c_{Q_i}, \, \mathbf{W}_q \rightarrow \mathbf{U}_q, \quad \mathbf{x}_q \equiv \frac{\text{diag}(m_{q_1}, m_{q_2}, m_{q_3})}{M_{\text{KK}}}$$

## Effects due to fermion mixing\*:

- mixing matrices  $\delta_A$  are parametrically of same order as  $\Delta_A$  since they are not suppressed by  $v^2/M_{\text{KK}}^2$  in Feynman rules
- fermion mixing is only source of flavor-breaking in Higgs-boson couplings



# Mixing matrices: Scaling relations

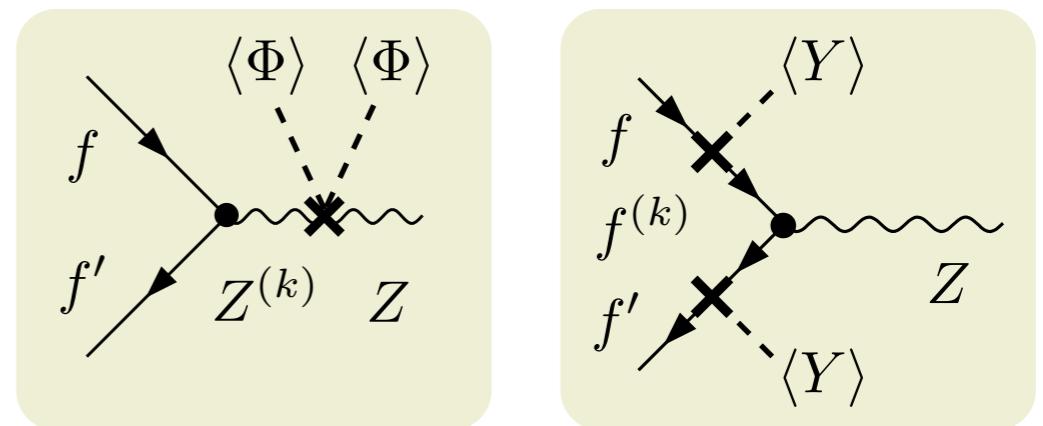
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$$(\Delta_Q^{(')})_{ij} \sim F_{c_{Q_i}} F_{c_{Q_j}}, \quad (\delta_Q)_{ij} \sim \frac{m_{q_i} m_{q_j}}{M_{KK}^2} \frac{1}{F_{c_{q_i}} F_{c_{q_j}}} \sim \frac{v^2 Y_q^2}{M_{KK}^2} F_{c_{q_i}} F_{c_{q_j}},$$

$$(\Delta_q^{(')})_{ij} \sim F_{c_{q_i}} F_{c_{q_j}}, \quad (\delta_q)_{ij} \sim \frac{m_{q_i} m_{q_j}}{M_{KK}^2} \frac{1}{F_{c_{Q_i}} F_{c_{Q_j}}} \sim \frac{v^2 Y_q^2}{M_{KK}^2} F_{c_{Q_i}} F_{c_{Q_j}}$$

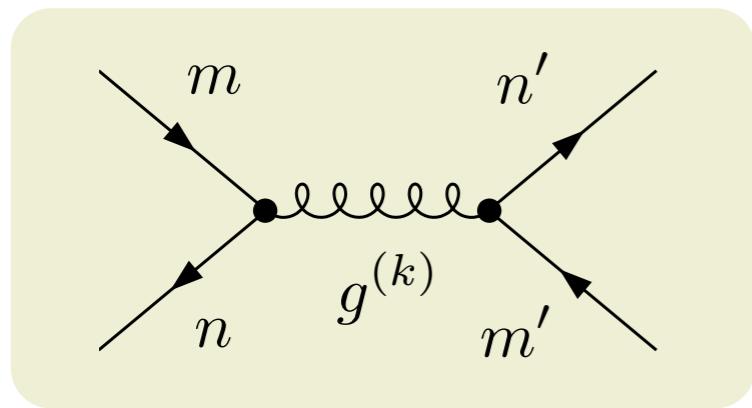
## Implications of scaling relations:

- ▶ all effects are proportional to  $F_{c_{A_i}} F_{c_{A_j}}$ , so that flavor-violating vertices involving UV-localized fermions are suppressed
- ▶ this suppression of dangerous FCNCs involving light quarks is referred to as RS-GIM mechanism\*

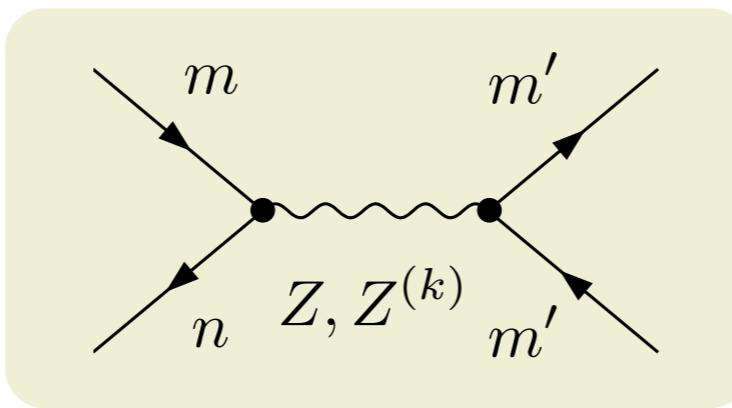


# Anatomy of tree-level FCNC processes

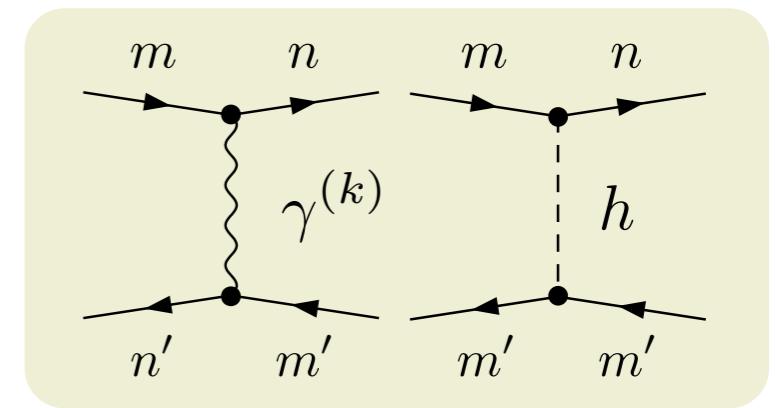
- Three types of generic contributions to dimension six operators:



dominant contribution to  
 $\Delta F = 2$  processes



dominant contribution to  
 $\Delta F = 1$  processes

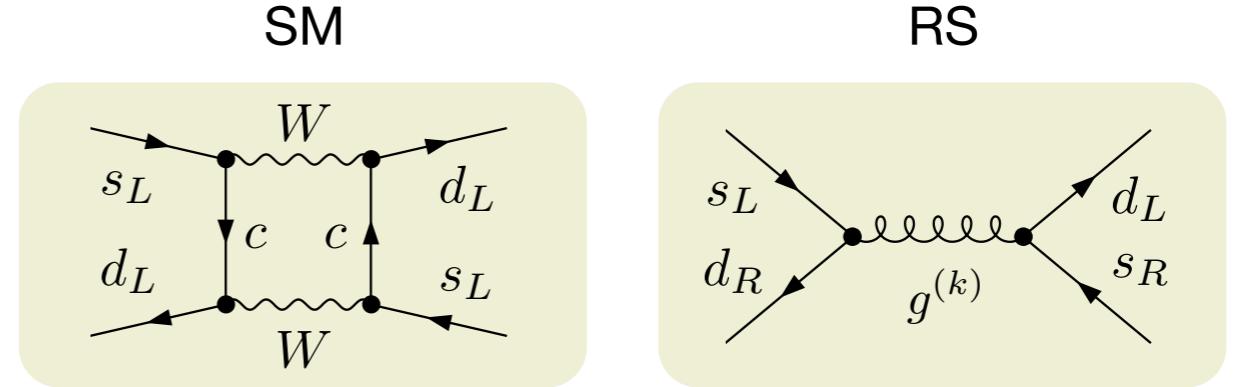


small contributions to  
 $\Delta F = 1, 2$  processes

- Like in SM, dimension five dipole-type operators contributing to  $B \rightarrow X_s \gamma$  or  $\mu \rightarrow e \gamma$  arise first at one-loop level

# Meson mixing: Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \sum_{i=1}^5 C_i Q_i + \sum_{i=1}^3 \tilde{C}_i \tilde{Q}_i$$



$$Q_1 = (\bar{d}_L^a \gamma_\mu s_L^a)(\bar{d}_L^b \gamma^\mu s_L^b),$$

$$Q_2 = (\bar{d}_R^a s_L^a)(\bar{d}_R^b s_L^b),$$

$$Q_3 = (\bar{d}_R^a s_L^b)(\bar{d}_R^b s_L^a),$$

$$Q_4 = (\bar{d}_R^a s_L^a)(\bar{d}_L^b s_R^b),$$

$$Q_5 = (\bar{d}_R^a s_L^b)(\bar{d}_L^b s_R^a),$$

$$\tilde{Q}_{1,2,3} : L \leftrightarrow R$$

$$C_1^{\text{RS}} = \frac{4\pi L}{M_{\text{KK}}^2} (\tilde{\Delta}_D)_{12} \otimes (\tilde{\Delta}_D)_{12} \left[ \frac{\alpha_s}{3} + 1.12\alpha \right],$$

$$\tilde{C}_1^{\text{RS}} = \frac{4\pi L}{M_{\text{KK}}^2} (\tilde{\Delta}_d)_{12} \otimes (\tilde{\Delta}_d)_{12} \left[ \frac{\alpha_s}{3} + 0.14\alpha \right],$$

$$C_4^{\text{RS}} = \frac{4\pi L}{M_{\text{KK}}^2} (\tilde{\Delta}_D)_{12} \otimes (\tilde{\Delta}_d)_{12} [-2\alpha_s],$$

$$C_5^{\text{RS}} = \frac{4\pi L}{M_{\text{KK}}^2} (\tilde{\Delta}_D)_{12} \otimes (\tilde{\Delta}_d)_{12} \left[ \frac{2\alpha_s}{3} - 0.29\alpha \right],$$

$$(\tilde{\Delta}_A)_{mn} \otimes (\tilde{\Delta}_B)_{m'n'} \rightarrow (\Delta_A)_{mn} (\Delta_B)_{m'n'}$$

# Meson mixing: Neutral kaons<sup>\*</sup>

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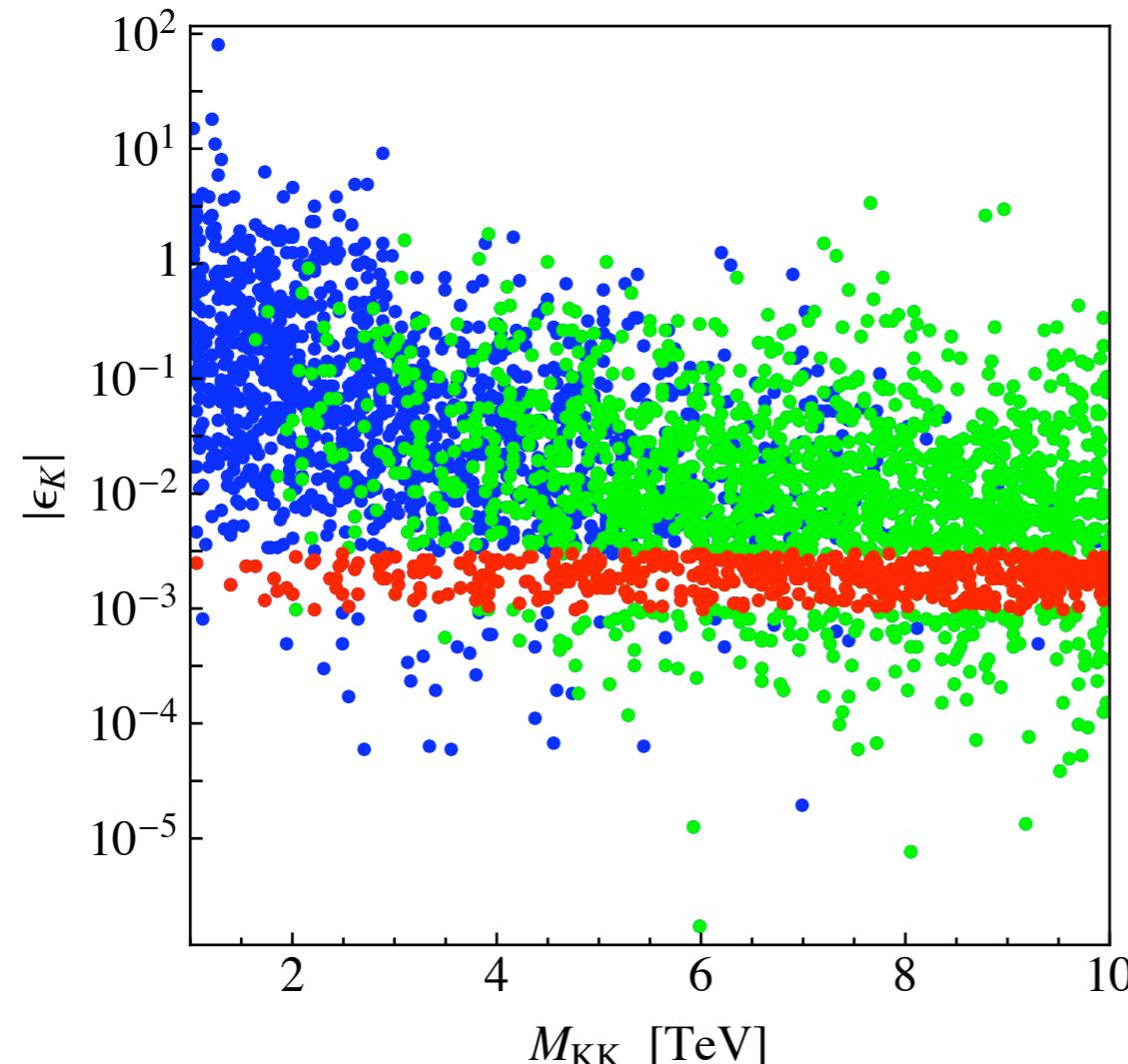
- Presence of tree-level FCNCs mediated by vector bosons generically leads to disastrous effects. Bounds on  $\Delta F = 2$  Wilson coefficients allow for sanity check in any BSM model
- In RS scenario model-independent limit on  $\text{Im } C_4$  following from  $\varepsilon_K$  imply that KK gluon mass has to be generically larger than 20 TeV
- Reason for stringent limit is enhancement of matrix element of  $Q_4$  by renormalization group evolution and chiral factor  $(m_K/m_s)^2$

In RS model:

$$\text{Re}(\varepsilon_K)_{\text{RS}} \approx -\frac{3.8 \cdot 10^{-3} \text{Im}[(\Delta_D)_{12}(\Delta_d)_{12} - 1.4 \cdot 10^{-3} ((\Delta_D)_{12}^2 + (\Delta_d)_{12}^2)]}{10^{-12} M_{\text{KK}}^2 \text{TeV}^{-2}}$$

# Meson mixing: Neutral kaons\*

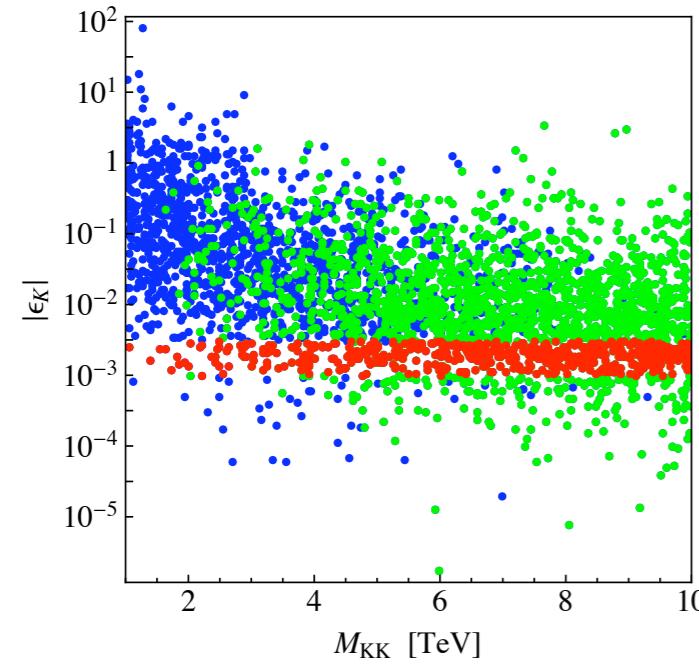
- Generically  $|\varepsilon_K| \approx 100 |\varepsilon_K|_{\text{exp}}$  in RS model where  $|\varepsilon_K|_{\text{exp}} = (2.23 \pm 0.01) \cdot 10^{-3}$ .  
But  $|\varepsilon_K| \approx |\varepsilon_K|_{\text{exp}}$  possible for low KK mass scale with some fine-tuning



3000 randomly chosen RS points with  $|Y_q| < 3$  reproducing quark masses and CKM parameters with  $\chi^2/\text{dof} < 11.5/10$  corresponding to 68% CL

- satisfying 95% CL limit  
 $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$
- without  $Z \rightarrow b\bar{b}$  constraint
- with  $Z \rightarrow b\bar{b}$  constraint at 95% CL

# Meson mixing: Ideas to solve $|\epsilon_K|$ problem\*

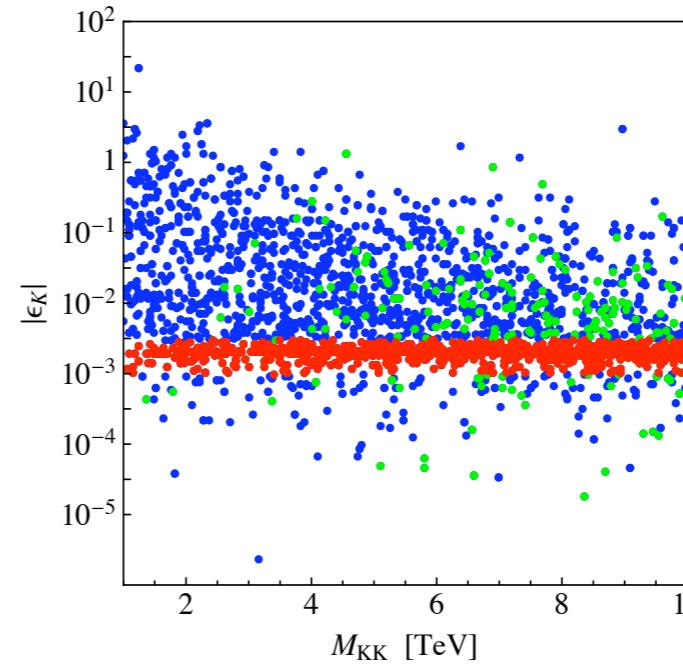


S1: Standard

$$|Y_q| < 3$$

- 16%
- 59%

13% pass

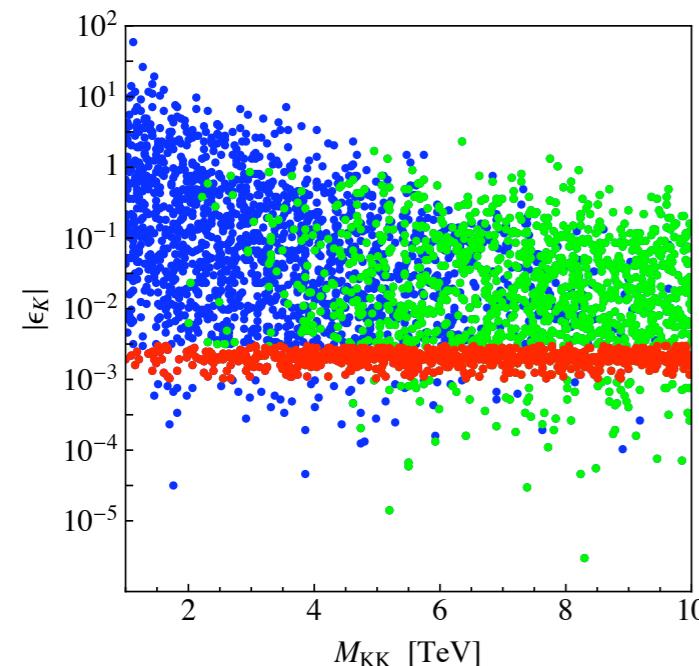


S2: Big Yukawas

$$|Y_q| < 12$$

- 44%
- 26%

19% pass

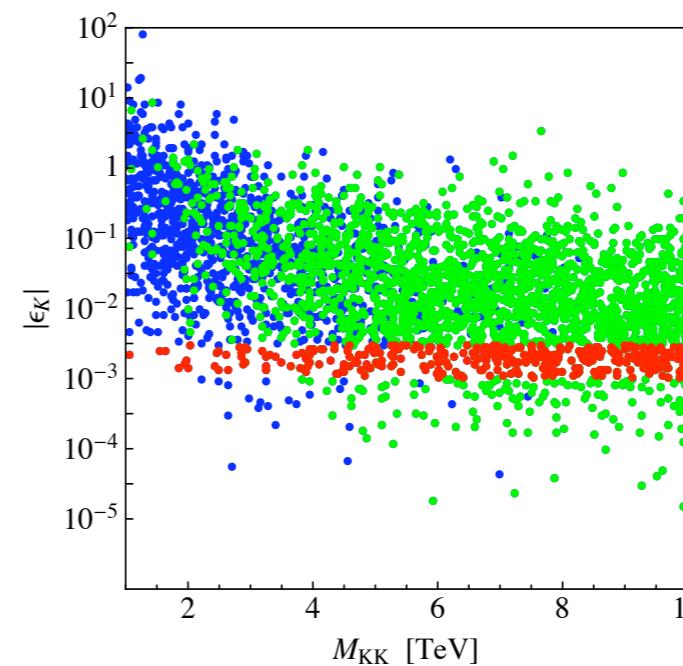


S3: Alignement

$$c_{d1} = c_{d2} = c_{d3}$$

- 48%
- 24%

16% pass



S4: Little RS

$$L = 7$$

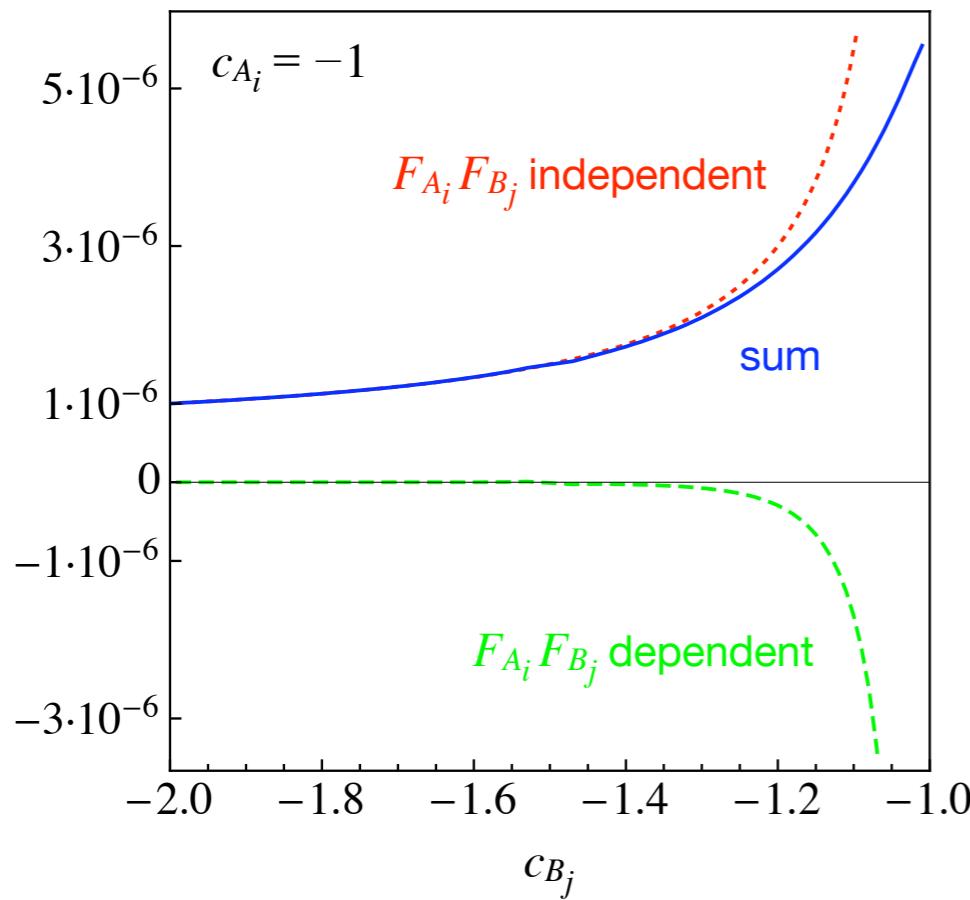
- 11%
- 68%

9% pass

\*Davoudiasl *et al.*, arXiv:0802.0203; Santiago, arXiv:0806.1230; Bauer *et al.*, arXiv:08xx.xxxx

# $|\varepsilon_K|$ in little RS models\*

- Since many amplitudes in RS model are enhanced by logarithm of warp factor  $L$  harmful effects can naively be suppressed by volume truncation



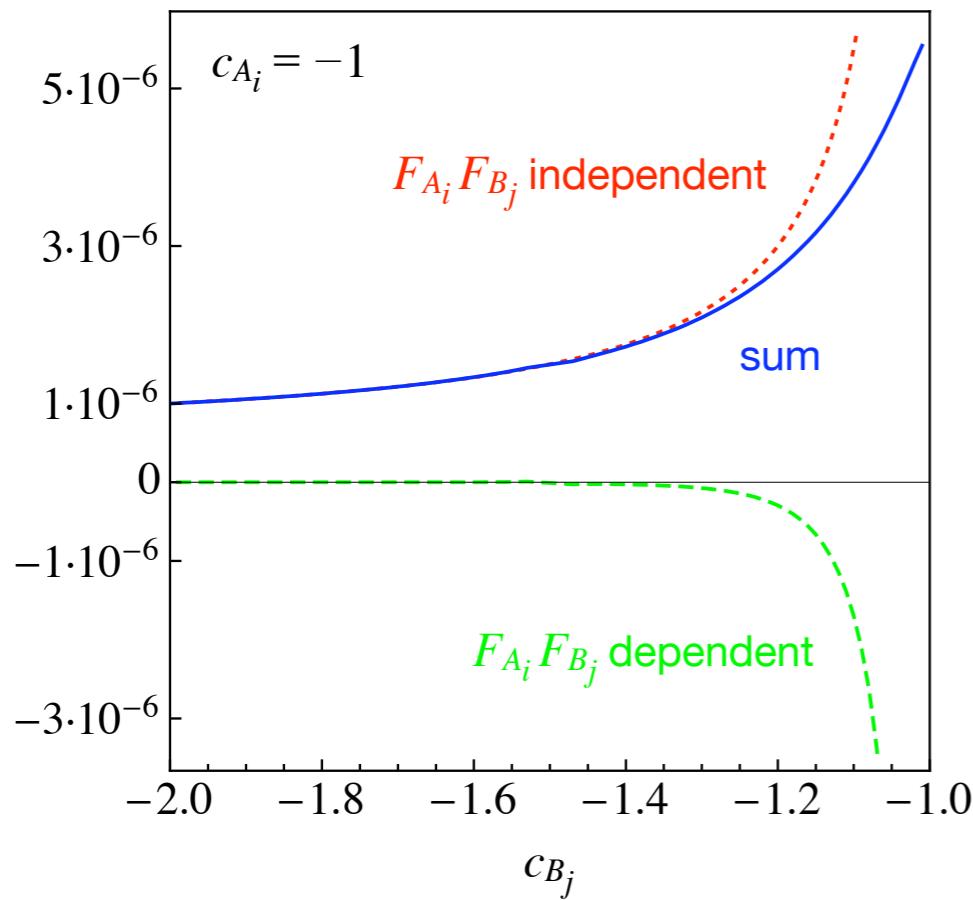
Typical bulk parameters for  $L = 7$ :

$$\begin{aligned}c_{Q_1} &= -1.06, & c_{Q_2} &= -0.77, & c_{Q_3} &= -0.61, \\c_{u_1} &= -1.92, & c_{u_2} &= -0.96, & c_{u_3} &= +0.34, \\c_{d_1} &= -1.75, & c_{d_2} &= -1.53, & c_{d_3} &= -0.93\end{aligned}$$

- For  $c_{A_i} + c_{B_j} < -2$  weight factor  $t_<^2$  appearing in overlap integrals of  $\tilde{\Delta}_A \otimes \tilde{\Delta}_B$  not sufficient to suppress light quark profiles in UV. This leads to breakdown of RS-GIM

# $|\varepsilon_K|$ in little RS models\*

- Since many amplitudes in RS model are enhanced by logarithm of warp factor  $L$  harmful effects can naively be suppressed by volume truncation



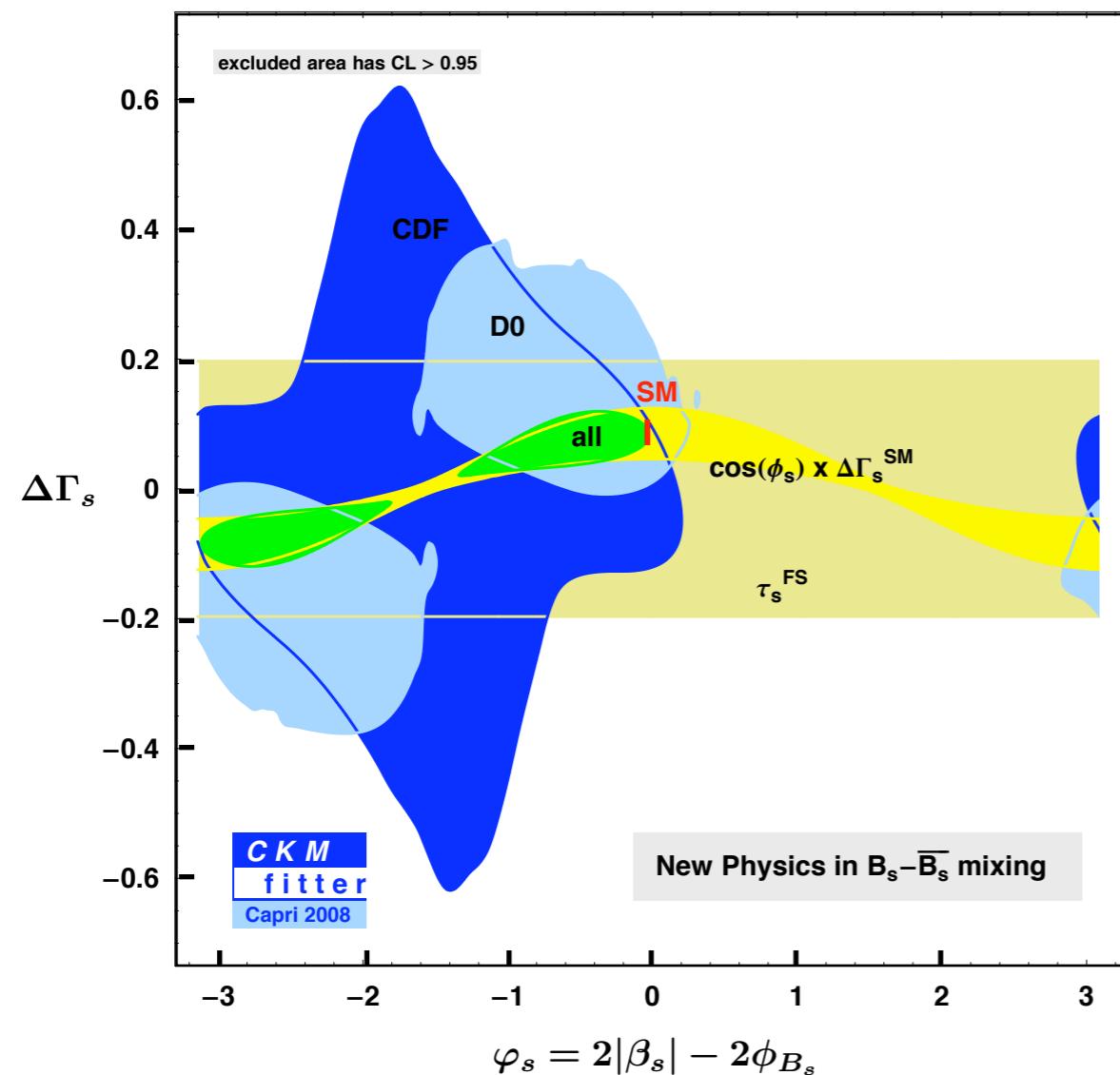
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- Barring cancellations,  $c_{A_i} + c_{B_j} > -2$  implies  $L \gtrsim 10$ ,  $L \gtrsim 11$ , and  $L \gtrsim 12$  in  $L \otimes L$ ,  $L \otimes R$ , and  $R \otimes R$  sector. UV-dominance in  $|\varepsilon_K|$  is thus natural feature of little RS models

# BSM physics in $B_s$ mixing\*

- Tantalizing hints for new physics phase in  $B_s - \bar{B}_s$  mixing from flavor-tagged analysis of mixing-induced CP violation in  $B_s \rightarrow J/\psi\phi$  by CDF and DØ



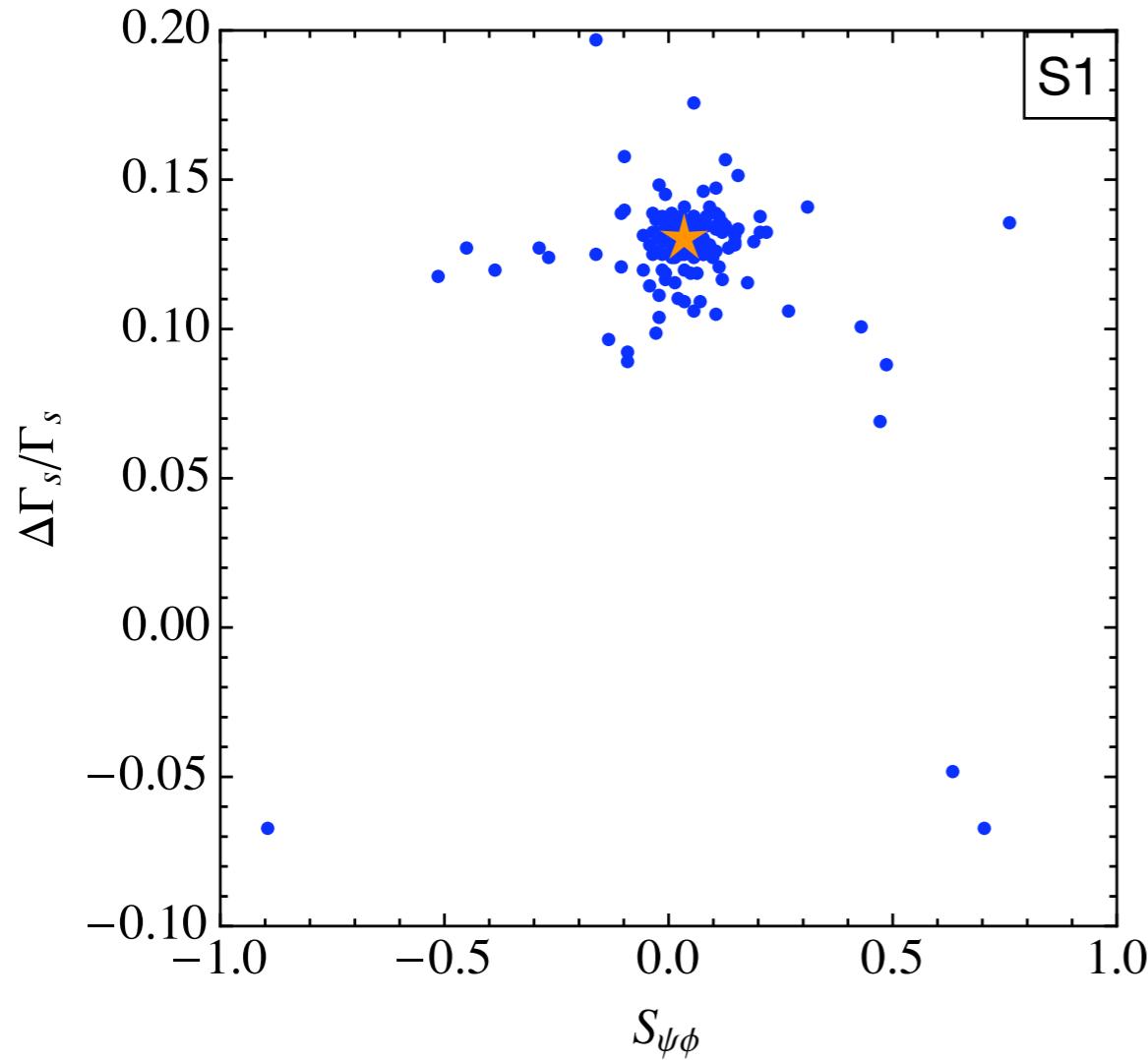
## CKMfitter combination:

- ▶ CDF data only  $2.1\sigma$
- ▶ DØ data only  $1.9\sigma$
- ▶ CDF and DØ data  $2.7\sigma$
- ▶ full BSM physics fit  $2.5\sigma$

Discrepancy of  $\varphi_s = 2|\beta_s| - 2\phi_{B_s}$  with respect to SM value  $\varphi_s \approx 2^\circ$  at around  $2\sigma$  level. Issue will be clarified at LHCb

# Meson mixing: Neutral $B_s$ mesons\*

- Constraint from  $|\varepsilon_K|$  does not exclude order one effects in width difference  $\Delta\Gamma_s/\Gamma_s$  of  $B_s$  system



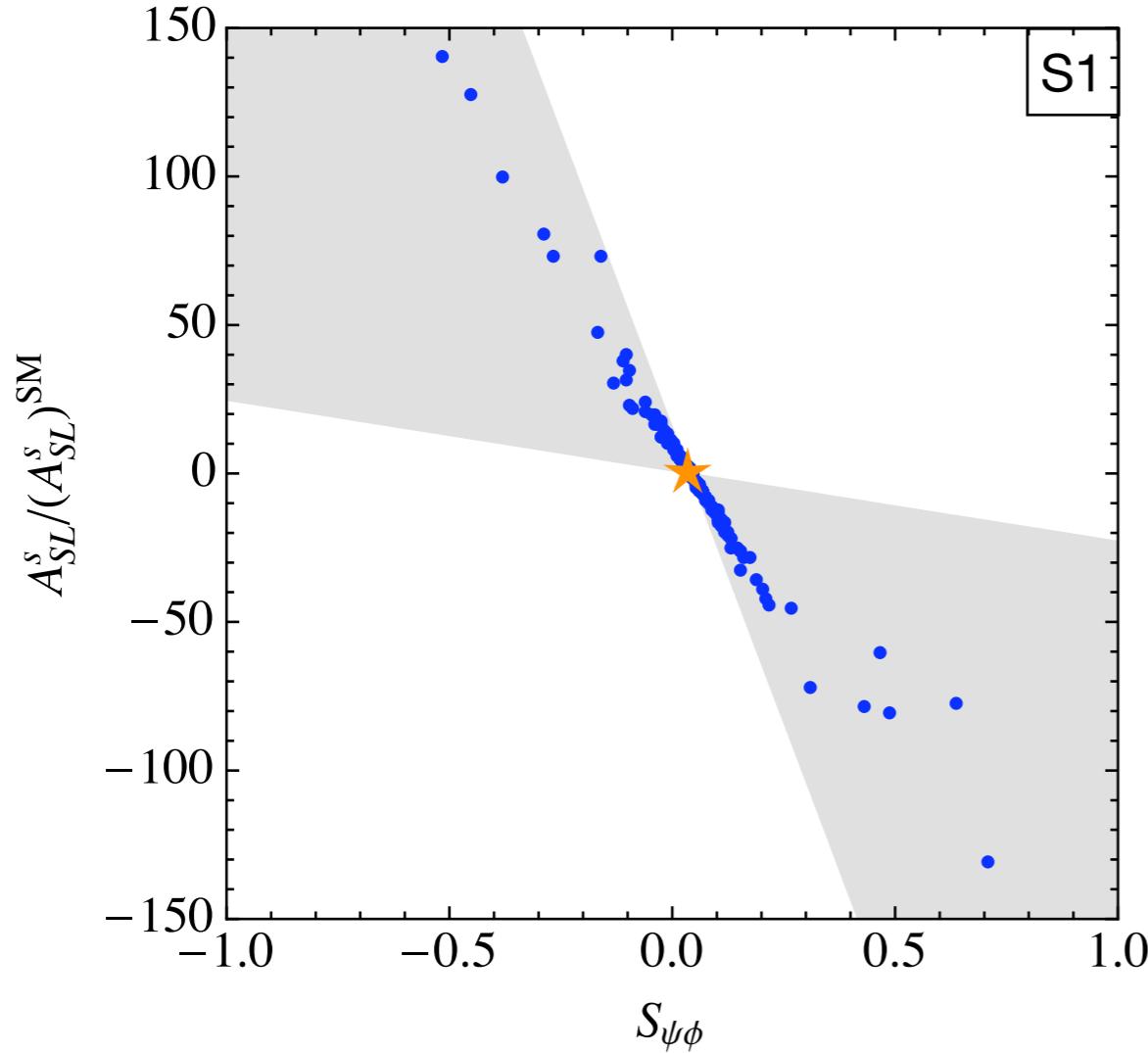
$$\begin{aligned}\Delta\Gamma_s &= \Gamma_L^s - \Gamma_S^s \\ &= 2 |\Gamma_{12}^s| \cos(2|\beta_s| - 2\phi_{B_s})\end{aligned}$$

★ SM:  $\Delta\Gamma_s/\Gamma_s \approx 0.13$ ,  $S_{\psi\phi} \approx 0.04$

- consistent with quark masses, CKM parameters, and 95% CL limit  $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

# Meson mixing: Neutral $B_s$ mesons\*

- In RS model significant, corrections to semileptonic CP asymmetry  $A_{SL}^s$  and  $S_{\psi\phi} = \sin(2|\beta_s| - 2\phi_{B_s})$  consistent with  $|\varepsilon_K|$  can arise



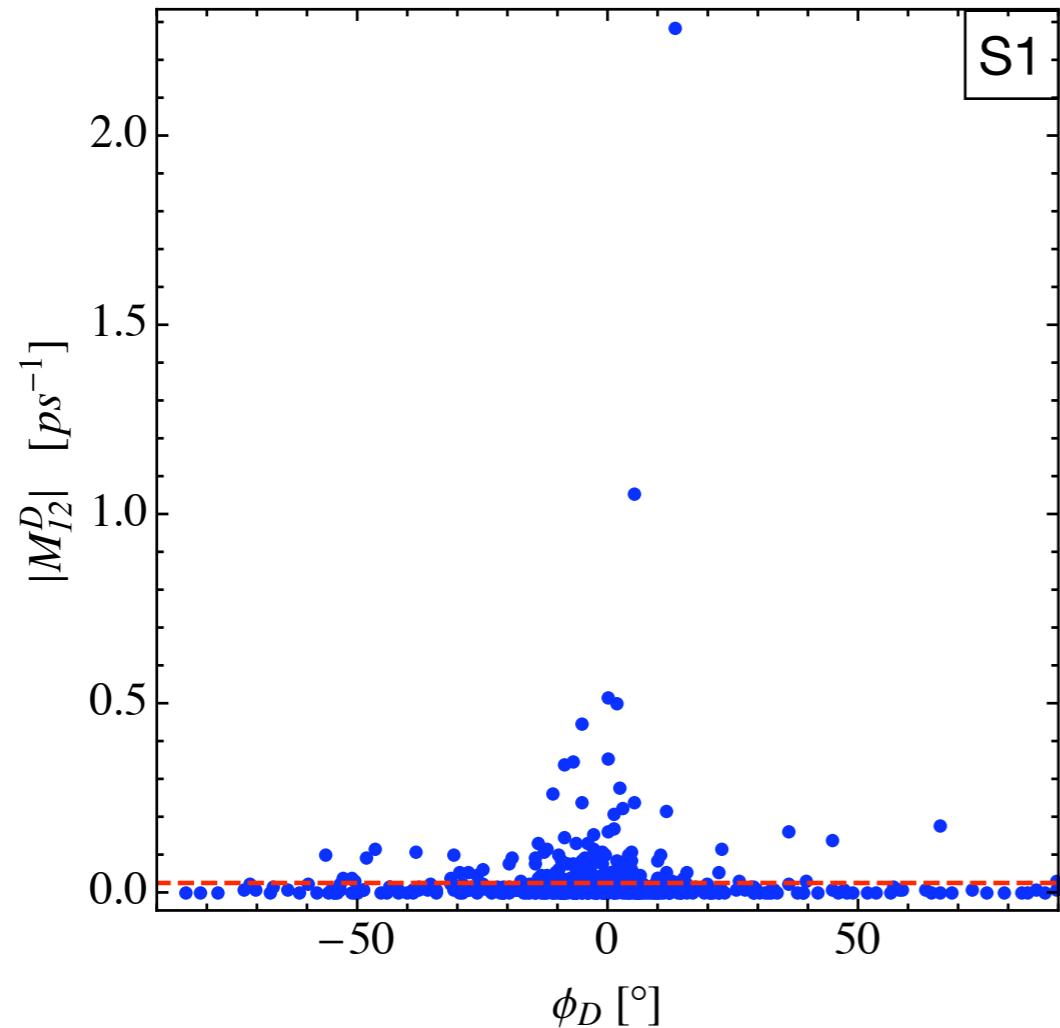
$$\begin{aligned} A_{SL}^s &= \frac{\Gamma(\bar{B}_s \rightarrow l^+ X) - \Gamma(B_s \rightarrow l^- X)}{\Gamma(\bar{B}_s \rightarrow l^+ X) + \Gamma(B_s \rightarrow l^- X)} \\ &= \text{Im} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right) \end{aligned}$$

★ SM:  $A_{SL}^s \approx 2 \cdot 10^{-5}$ ,  $S_{\psi\phi} \approx 0.04$

- model-independent prediction
- consistent with quark masses, CKM parameters, and 95% CL limit  $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

# Meson mixing: Neutral $D$ mesons\*

- Very large effects possible in  $D - \bar{D}$  mixing, including large CP violation.  
Prediction might be testable at LHCb

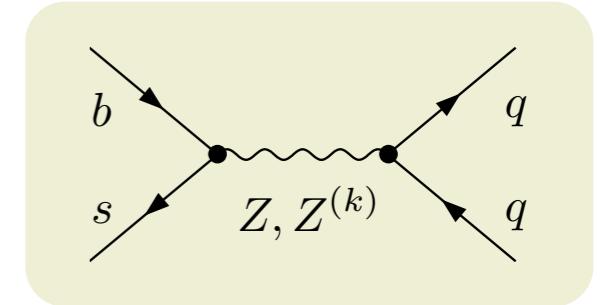
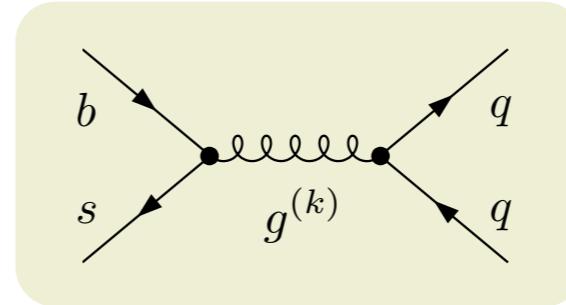


$$\begin{aligned}(M_{12}^D)^* &= \langle \bar{D} | \mathcal{H}_{\text{eff}, \text{RS}}^{\Delta C=2} | D \rangle \\ &= |M_{12}^D| e^{2i\phi_D}\end{aligned}$$

- maximal allowed SM effect with no significant CP phase
- consistent with quark masses, CKM parameters, and 95% CL limit  $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

# Rare decays: Effective Hamiltonian\*

$$\mathcal{H}_{\text{eff,RS}}^{b \rightarrow s q \bar{q}} = \sum_{i=3}^{10} \left( C_i^{\text{RS}} Q_i + \tilde{C}_i^{\text{RS}} \tilde{Q}_i \right)$$



$$Q_3 = 4 (\bar{s}_L^a \gamma^\mu b_L^a) \sum_q (\bar{q}_L^b \gamma_\mu q_L^b),$$

⋮

$$Q_6 = 4 (\bar{s}_L^a \gamma^\mu b_L^b) \sum_q (\bar{q}_R^b \gamma_\mu q_R^a),$$

$$Q_7 = 6 (\bar{s}_L^a \gamma^\mu b_L^a) \sum_q Q_q (\bar{q}_R^b \gamma_\mu q_R^b),$$

⋮

$$Q_{10} = 6 (\bar{s}_L^a \gamma^\mu b_L^b) \sum_q Q_q (\bar{q}_L^b \gamma_\mu q_L^a),$$

$$\tilde{Q}_{3-10}: L \leftrightarrow R$$

- KK gluons give dominant contribution to QCD penguins  $Q_{3-6}$ . Electroweak penguins  $Q_{7-10}$  arise almost entirely from exchange of  $Z$  and its KK modes

# Rare decays: Effective Hamiltonian\*

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- Analogous expressions for Wilson coefficients  $\tilde{C}_{3-10}^{\text{RS}}$  of opposite-chirality operators

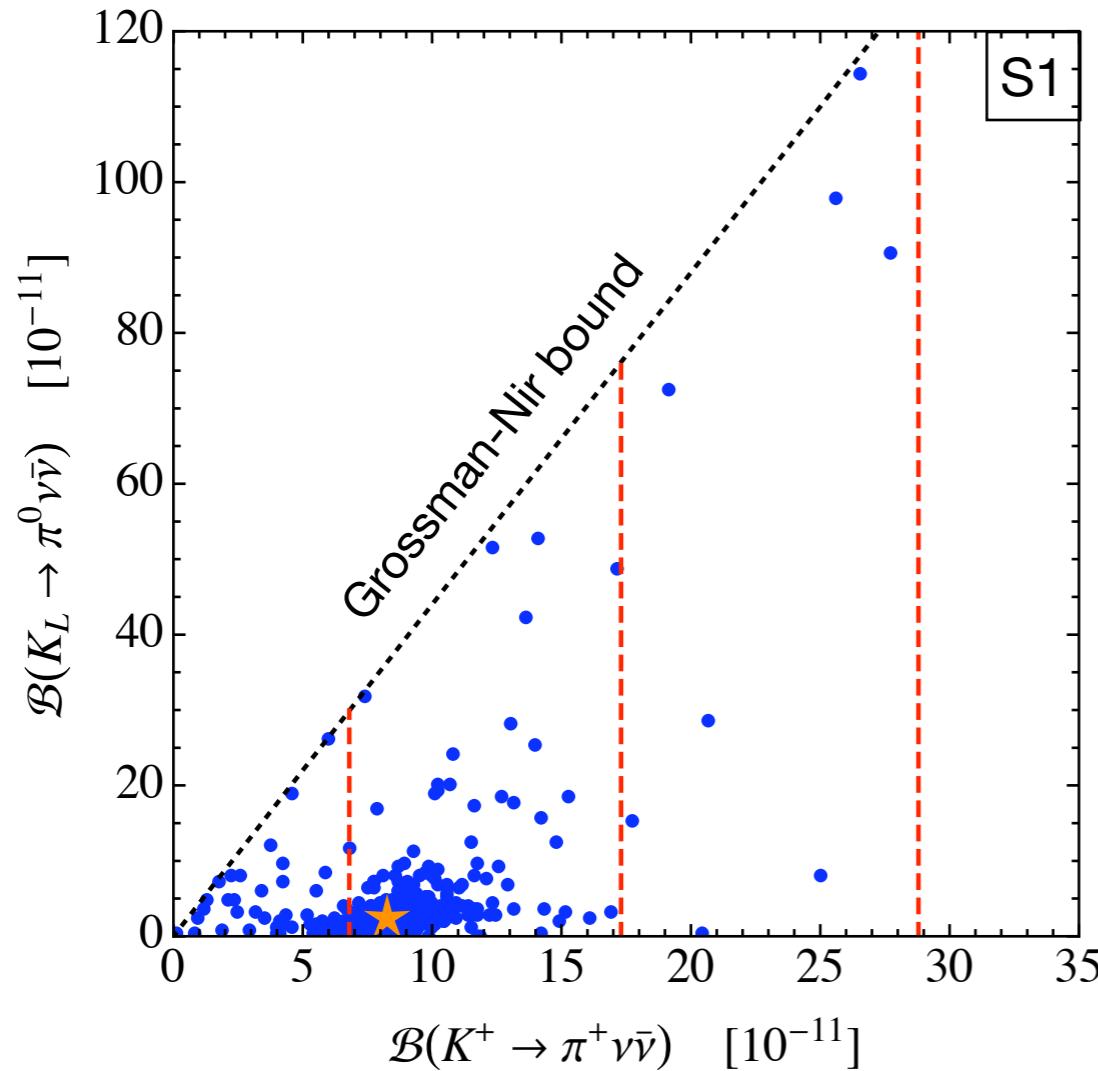
Only four couplings:

- $\Delta_Q, \Delta_q$  arising from  $g^{(k)}$ ,  $\gamma^{(k)}$  and  $\Sigma_Q, \Sigma_q$  due to  $Z$ ,  $Z^{(k)}$  exchange
- former two couplings can be made small, but latter ones cannot

$$\begin{aligned}
C_3^{\text{RS}} &= \frac{\pi\alpha_s}{M_{\text{KK}}^2} \frac{(\Delta_D)_{23}}{6} - \frac{\pi\alpha}{6s_w^2 c_w^2 M_{\text{KK}}^2} (\Sigma_D)_{23}, \\
C_4^{\text{RS}} = C_6^{\text{RS}} &= -\frac{\pi\alpha_s}{2M_{\text{KK}}^2} (\Delta_D)_{23}, \\
C_5^{\text{RS}} &= \frac{\pi\alpha_s}{6M_{\text{KK}}^2} (\Delta_D)_{23}, \\
C_7^{\text{RS}} &= \frac{2\pi\alpha}{9M_{\text{KK}}^2} (\Delta_D)_{23} - \frac{2\pi\alpha}{3c_w^2 M_{\text{KK}}^2} (\Sigma_D)_{23}, \\
C_8^{\text{RS}} = C_{10}^{\text{RS}} &= 0, \\
C_9^{\text{RS}} &= \frac{2\pi\alpha}{9M_{\text{KK}}^2} (\Delta_D)_{23} + \frac{2\pi\alpha}{3s_w^2 M_{\text{KK}}^2}, \\
\Sigma_Q &= L \left( \frac{1}{2} - \frac{s_w^2}{3} \right) \Delta'_Q + \frac{M_{\text{KK}}^2}{m_Z^2} \delta_Q
\end{aligned}$$

# Rare $K$ decays: Golden modes\*

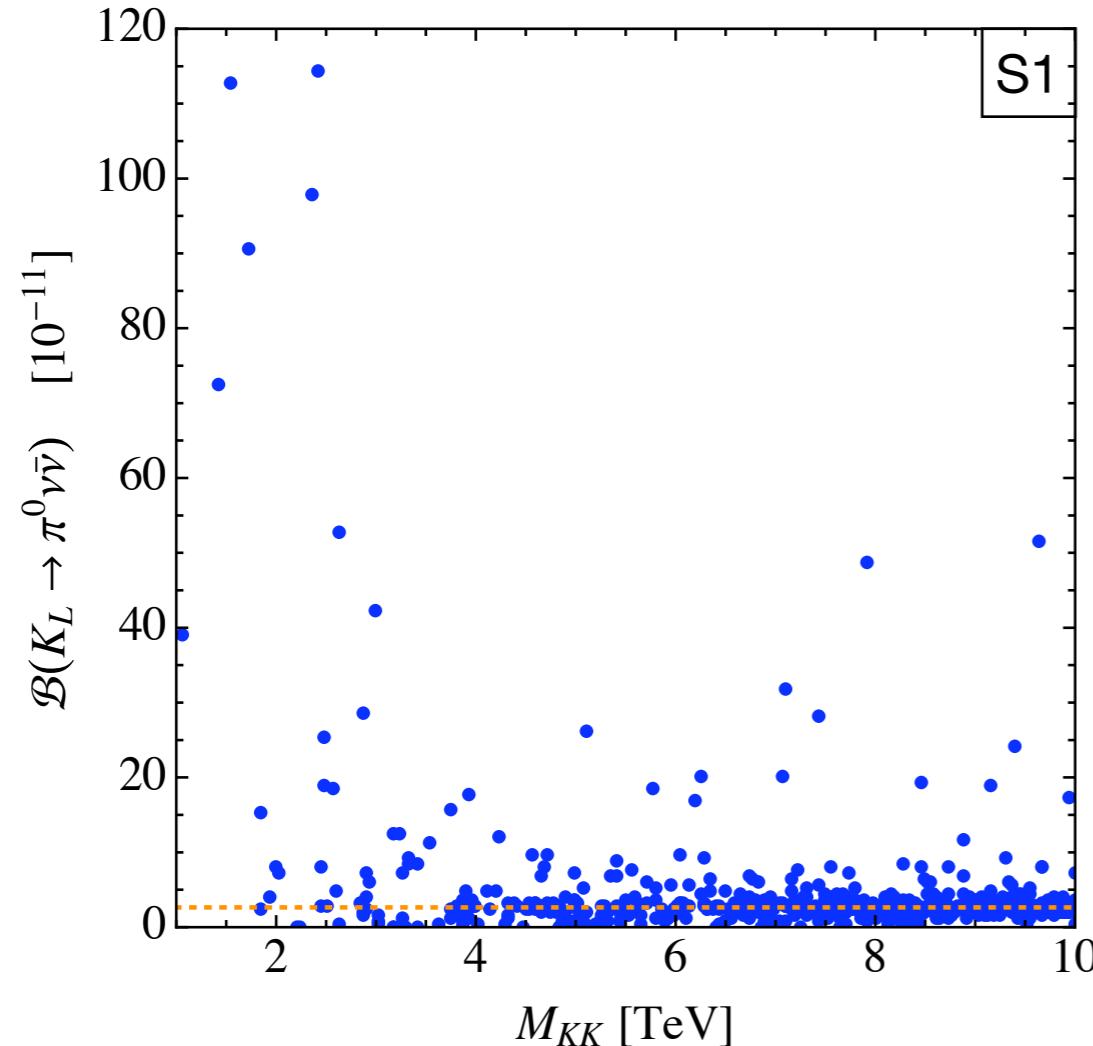
- Spectacular corrections in very clean  $K \rightarrow \pi v\bar{v}$  decays. Even Grossman-Nir bound,  $\mathcal{B}(K_L \rightarrow \pi^0 v\bar{v}) < 4.4 \mathcal{B}(K^+ \rightarrow \pi^+ v\bar{v})$ , can be saturated



- ★ SM:  $\mathcal{B}(K^+ \rightarrow \pi^+ v\bar{v}) \approx 8.3 \cdot 10^{-11}$ ,  
 $\mathcal{B}(K_L \rightarrow \pi^0 v\bar{v}) \approx 2.7 \cdot 10^{-11}$
- central value and 68% CL limit  
 $\mathcal{B}(K^+ \rightarrow \pi^+ v\bar{v}) = (17.3_{-10.5}^{+11.5}) \cdot 10^{-11}$   
from E949
- consistent with quark masses,  
CKM parameters, and 95% CL  
limit  $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

# Rare $K$ decays: Golden modes\*

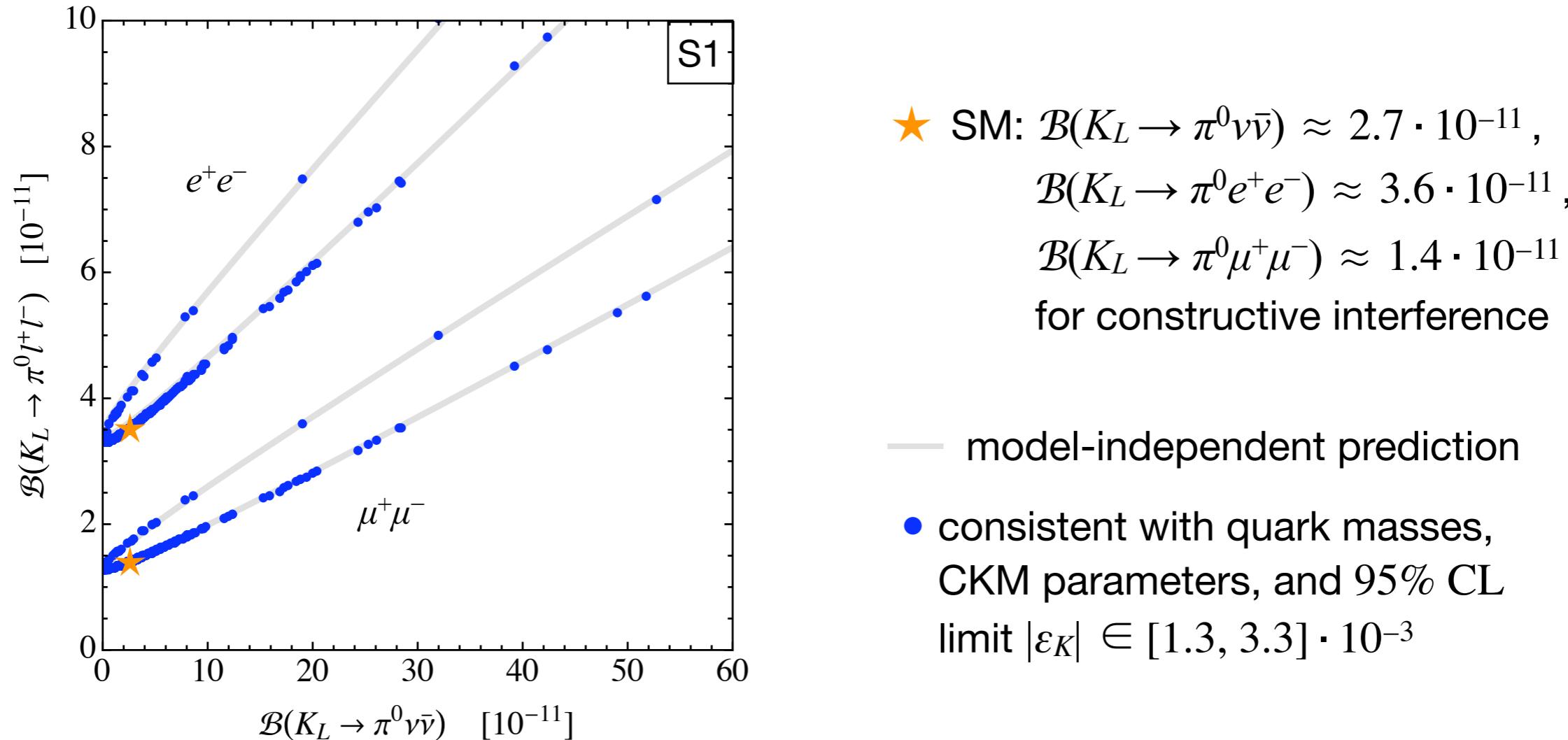
- Sensitivity to KK scale extends far beyond LHC reach.  $K \rightarrow \pi v\bar{v}$  modes offer unique window to BSM physics at and beyond terascale



- $m_{Z^{(1)}} \approx 2.50 M_{KK} ,$   
 $m_{Z^{(2)}} \approx 5.59 M_{KK} ,$   
⋮
- ..... SM:  $\mathcal{B}(K_L \rightarrow \pi^0 v\bar{v}) \approx 2.7 \cdot 10^{-11}$
- consistent with quark masses, CKM parameters, and 95% CL  
limit  $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

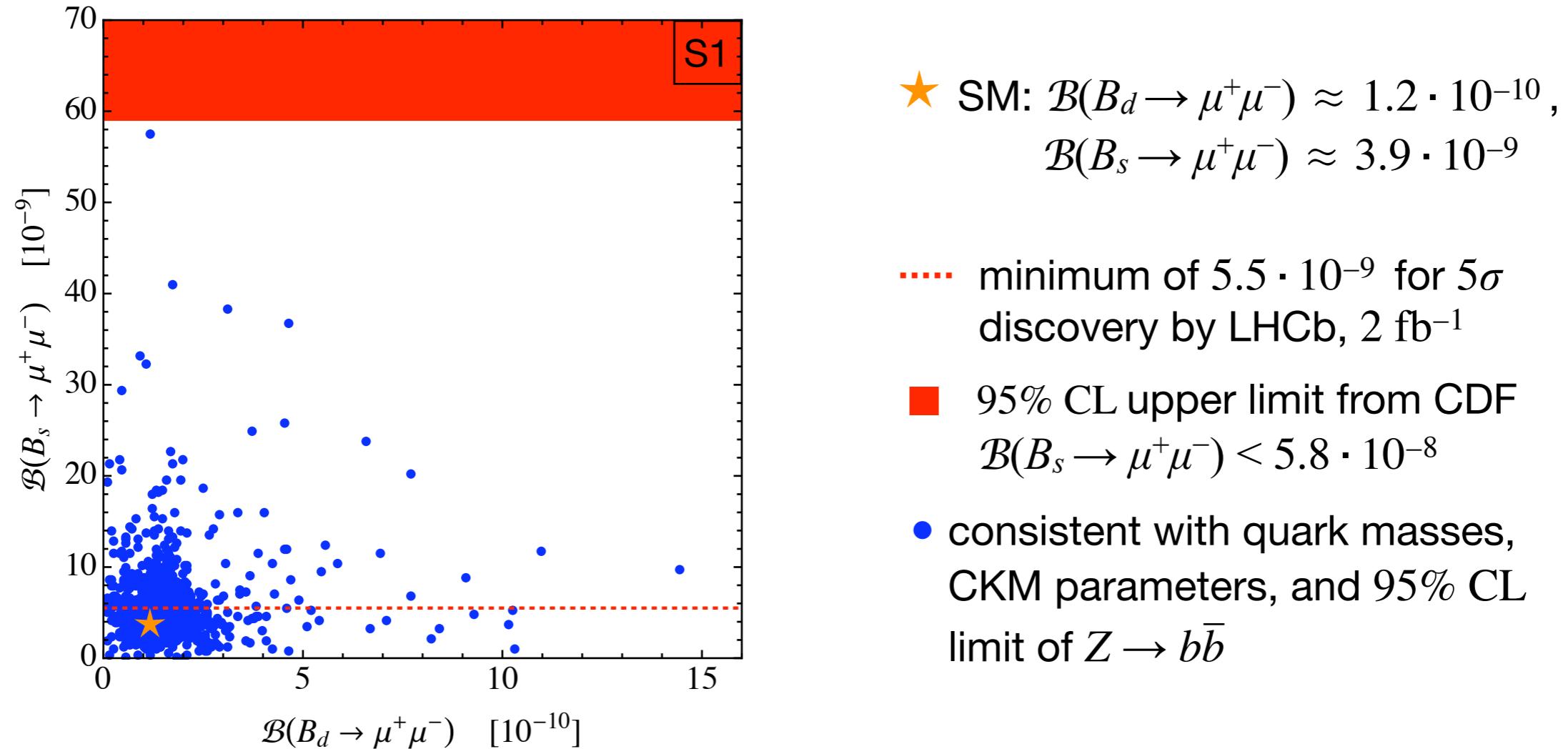
# Rare $K$ decays: Silver modes\*

- Deviations from SM expectations in  $K_L \rightarrow \pi^0 v\bar{v}$  and  $K_L \rightarrow \pi^0 l^+l^-$  follow specific pattern, arising from smallness of vector and scalar contributions



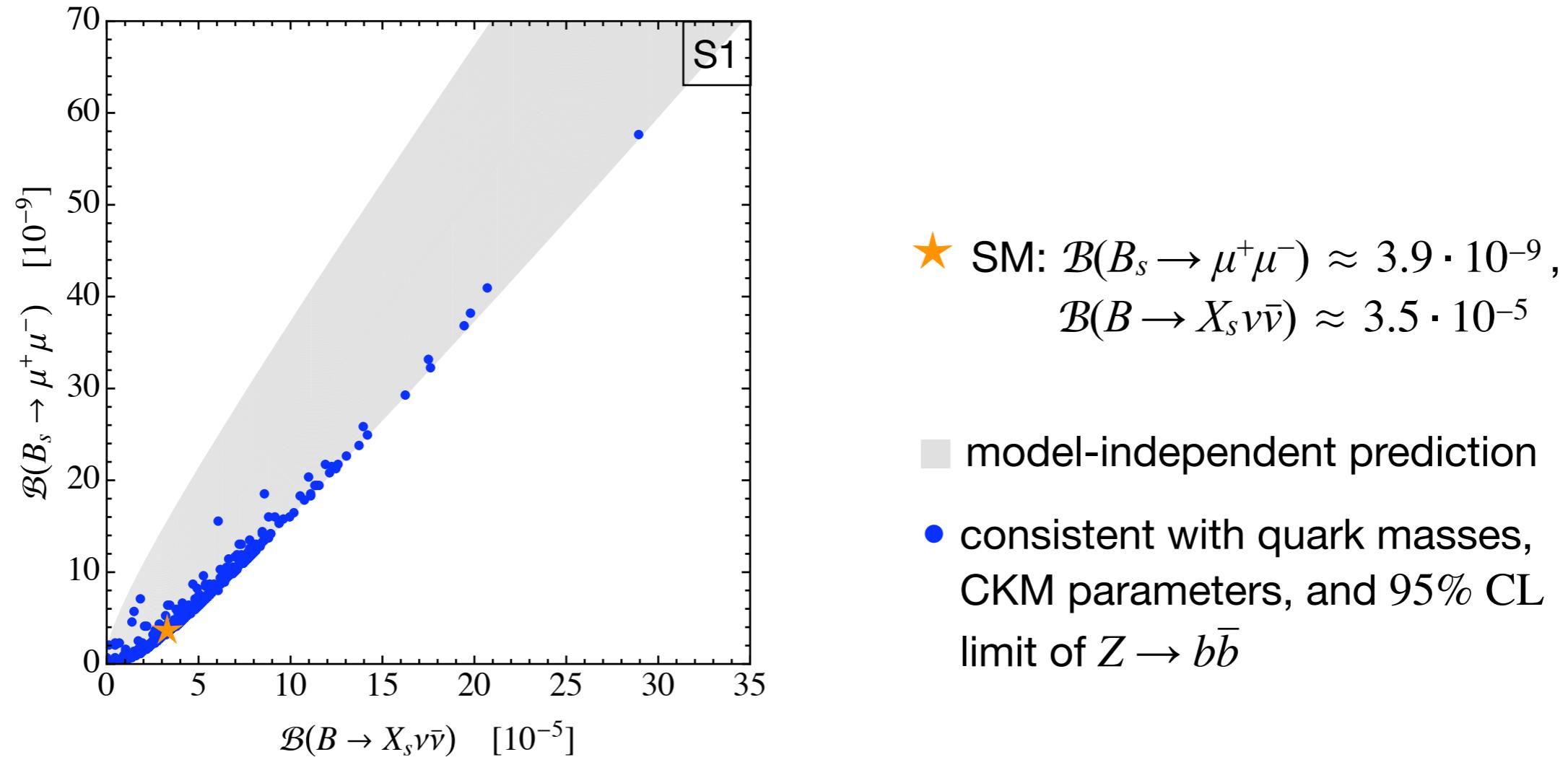
# Rare $B$ decays: Purely leptonic modes\*

- Factor ten enhancements possible in rare  $B_{d,s} \rightarrow \mu^+ \mu^-$  modes without violation of  $Z \rightarrow b\bar{b}$  constraints. Effects largely uncorrelated with  $|\varepsilon_K|$



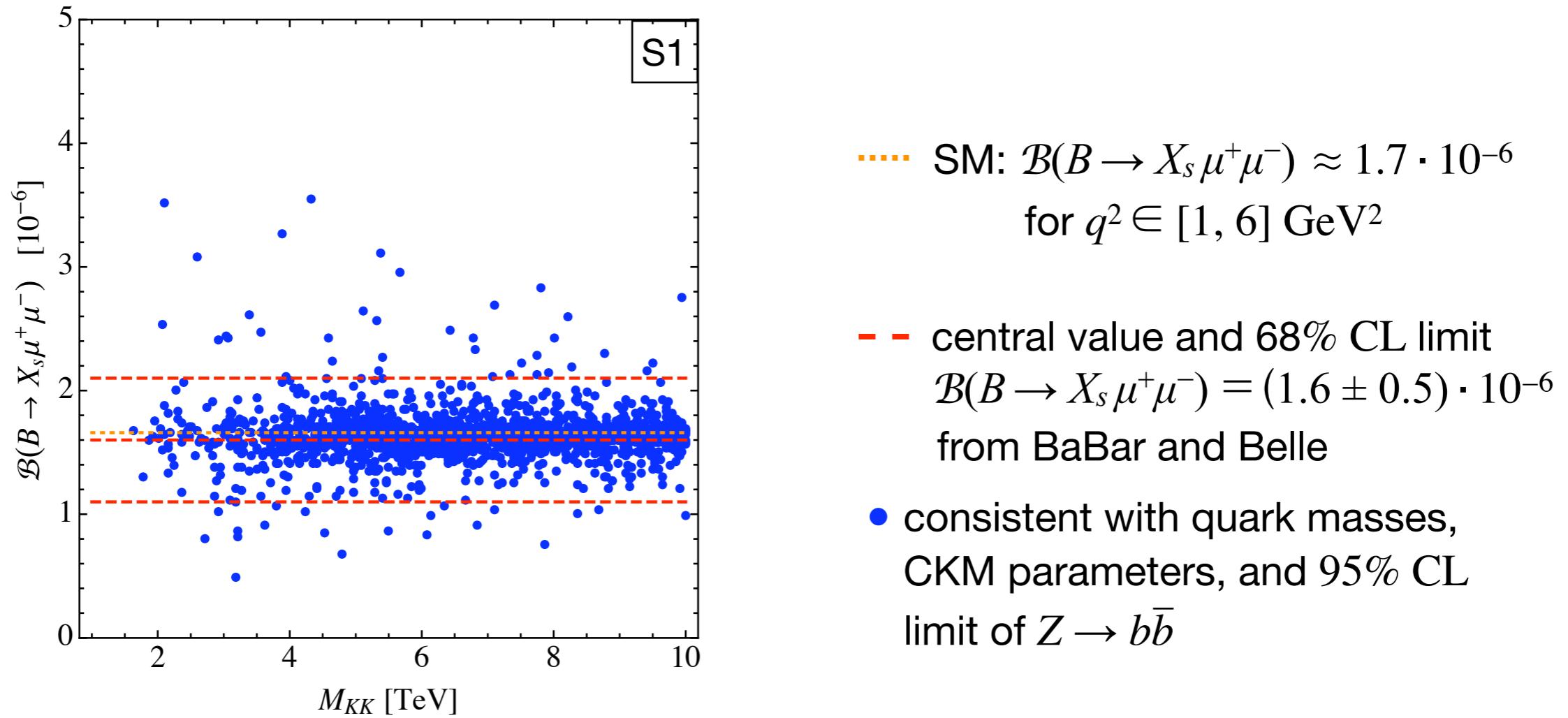
# Rare $B$ decays: Purely leptonic modes\*

- Enhancements in  $B_{d,s} \rightarrow \mu^+ \mu^-$  strongly correlated with ones in very rare decays  $B \rightarrow X_{d,s} \nu \bar{\nu}$ . Pattern again result of axial-vector dominance



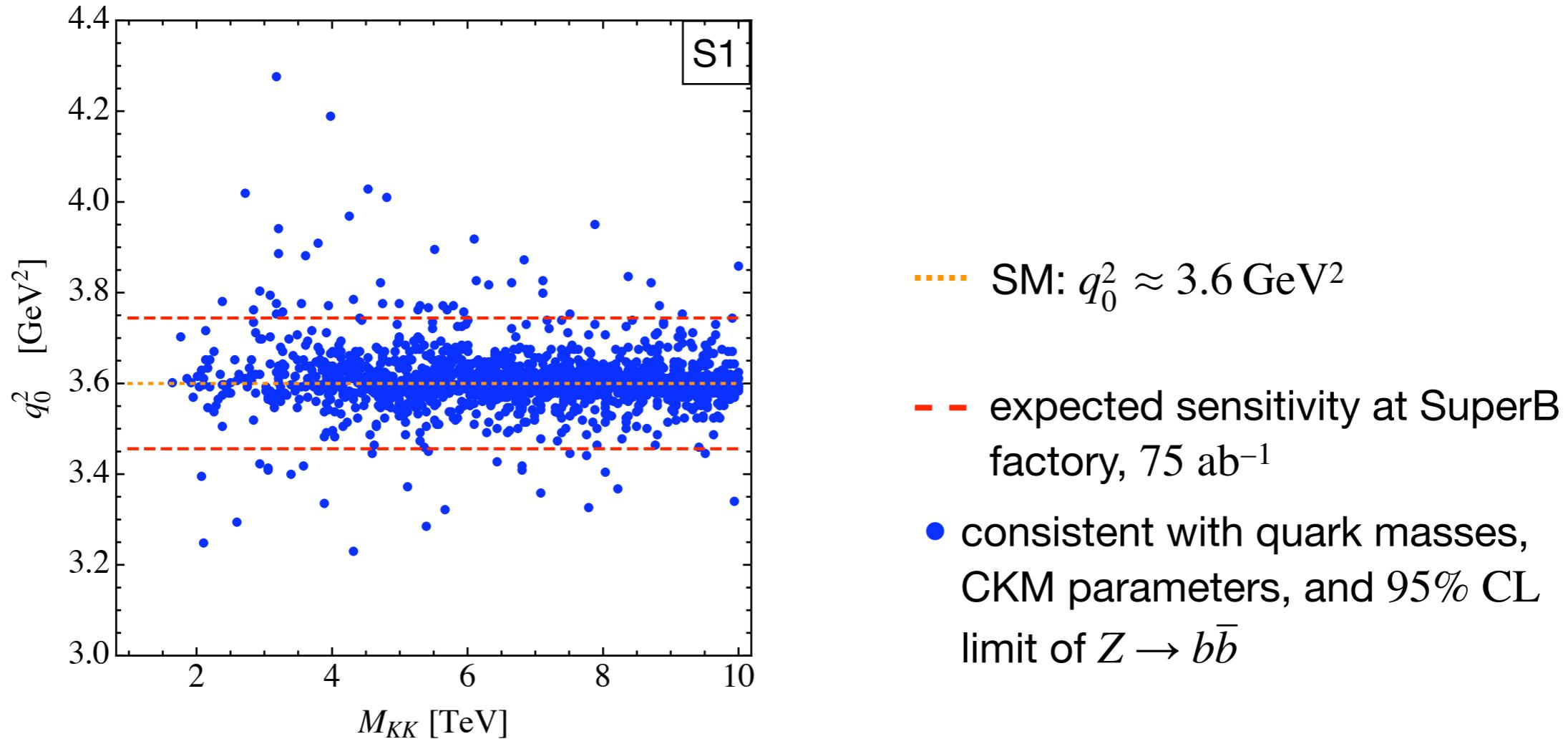
# Rare $B$ decays: Inclusive semileptonic modes\*

- Once  $Z \rightarrow b\bar{b}$  constraint is satisfied, values for  $B \rightarrow X_s \mu^+ \mu^-$  branching ratio arising from  $Z$  and  $Z^{(k)}$  exchange are typically within experimental limits



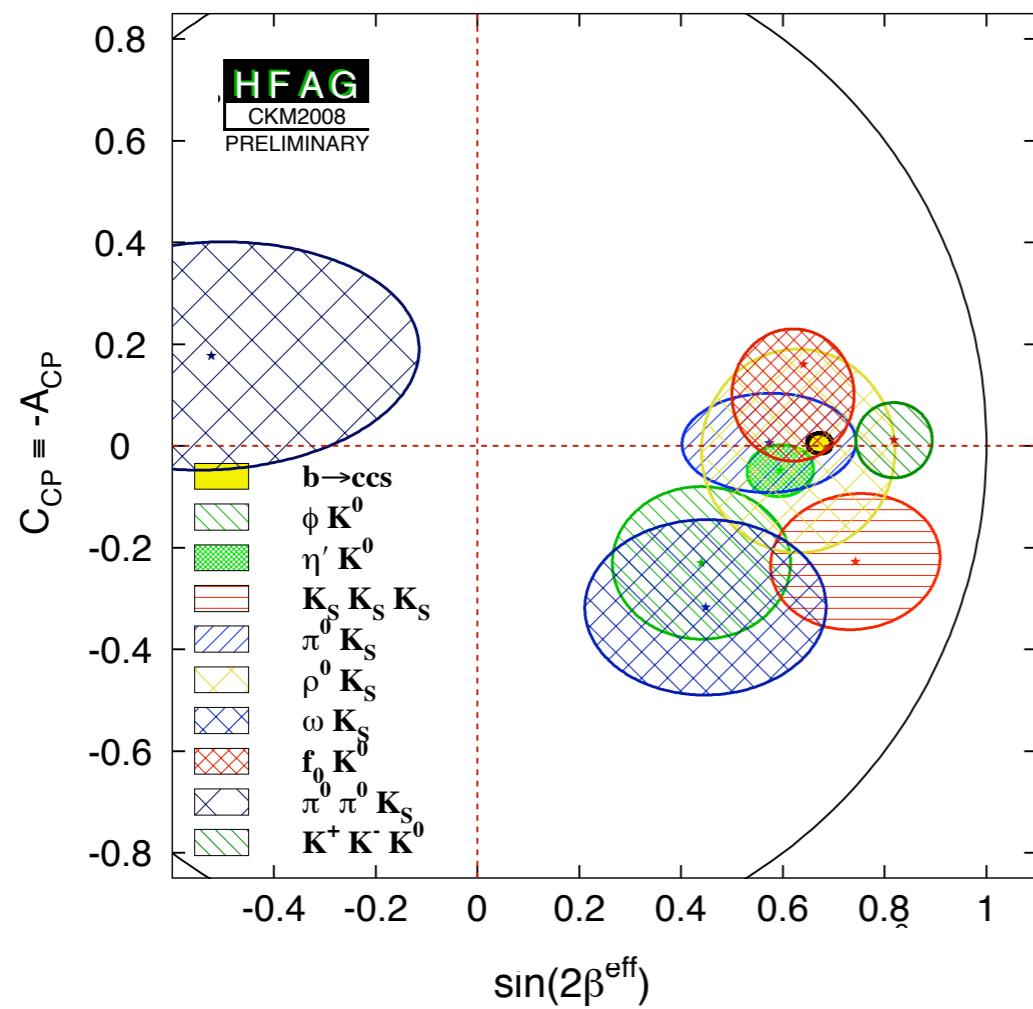
# Rare $B$ decays: Inclusive semileptonic modes\*

- Deviations of zero of forward-backward asymmetry,  $q_0^2$ , in  $B \rightarrow X_s \mu^+ \mu^-$  from SM prediction might be observable at high-luminosity flavor factory



# Non-leptonic $B$ and $K$ decays\*

- Electroweak penguin effects in rare hadronic decays such as  $B \rightarrow K\pi$  or  $B \rightarrow \phi K$  are naturally of order one compared to SM and can introduce new large CP-violating phases. Similar effects can occur in  $K \rightarrow \pi\pi$



## Potentially relevant for:

- ▶ explaining large CP asymmetries in  $B \rightarrow K\pi$  and determining of  $\sin(2\beta^{\text{eff}})$  from penguin-dominated modes
- ▶ studying of correlations between ratio  $\varepsilon'_K/\varepsilon_K$  measuring direct and indirect CP violation in  $K \rightarrow \pi\pi$  and large effects in rare  $K$  decays

# Conclusions

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- Models with warped extra dimension offer elegant solution to both gauge and fermion hierarchy problem
- Rich structure of flavor-violating interactions in gauge couplings to quarks generically not of CKM-type
- Mixing amplitudes dominated by KK gluon exchange, while rare decays receive largest contribution from diagrams involving  $Z$
- Effects naturally of order one or larger in modes where deviations from SM are allowed or indicated by data, while small in other modes
- Flavor-changing transitions of  $K$  and  $B_s$  mesons particularly interesting in RS framework

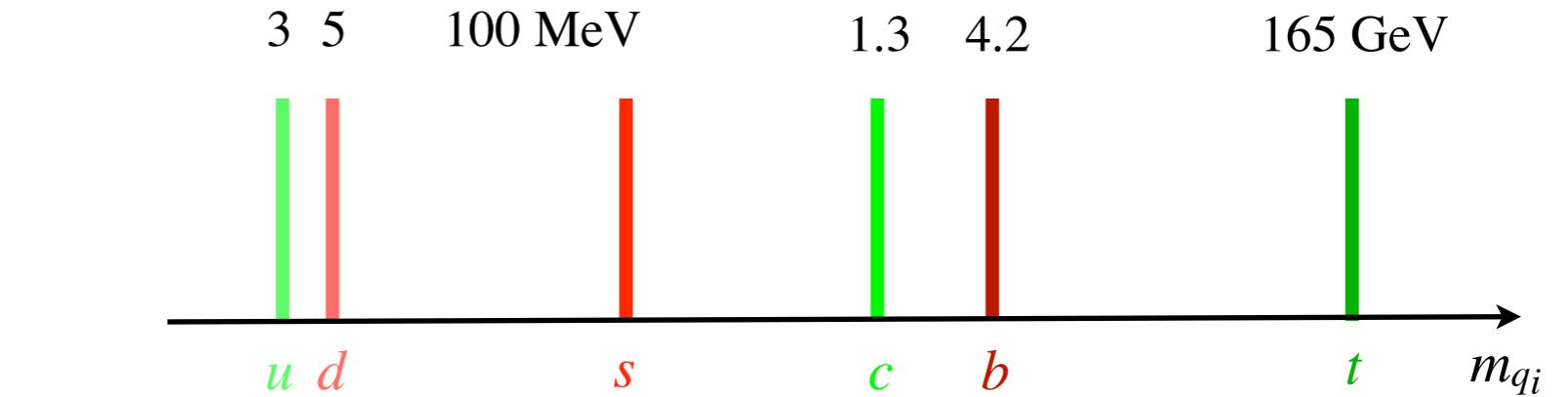
# Quark masses and mixings in RS model\*

$$m_{q_i} = \mathcal{O}(1) \frac{v}{\sqrt{2}} F_{c_{Q_i}} F_{c_{q_j}} ,$$

$$\lambda = \mathcal{O}(1) \frac{F_{c_{Q_1}}}{F_{c_{Q_2}}} ,$$

$$A = \mathcal{O}(1) \frac{F_{c_{Q_2}}^3}{F_{c_{Q_1}}^2 F_{c_{Q_3}}^2} ,$$

$$\bar{\rho} - i\bar{\eta} = \mathcal{O}(1)$$



$$V_{\text{CKM}} \approx \begin{pmatrix} 1 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda \approx 0.23, \quad A \approx 0.81, \quad \bar{\rho} \approx 0.14, \quad \bar{\eta} \approx 0.34$$

- Hierarchy in quark masses and mixings can be naturally generated from anarchic complex  $3 \times 3$  matrices  $Y_q = \mathcal{O}(1)$  entering  $Y_q^{\text{eff}} = F_{c_{Q_i}} (Y_q)_{ij} F_{c_{q_j}}$

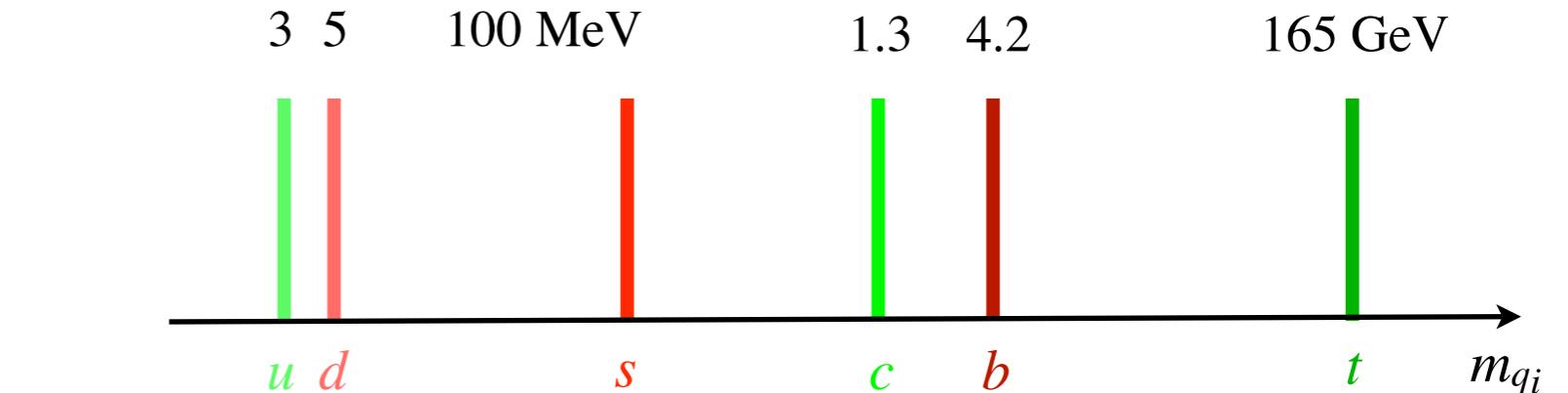
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$$A = \mathcal{O}(1) \frac{F_{c_{Q_2}}^3}{F_{c_{Q_1}}^2 F_{c_{Q_3}}^2} ,$$

$$\bar{\rho} - i\bar{\eta} = \mathcal{O}(1)$$



$$V_{\text{CKM}} \approx \begin{pmatrix} 1 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda \approx 0.23, \quad A \approx 0.81, \quad \bar{\rho} \approx 0.14, \quad \bar{\eta} \approx 0.34$$

- CKM angles fix hierarchy of  $F_{c_{Q_1}}/F_{c_{Q_2}} \sim \lambda$ ,  $F_{c_{Q_2}}/F_{c_{Q_3}} \sim \lambda^2$ ,  $F_{c_{Q_1}}/F_{c_{Q_3}} \sim \lambda^3$ , while quark masses determine  $F_{c_{u_1}}/F_{c_{u_2}} \sim m_u/m_t 1/\lambda^3$ ,  $F_{c_{u_2}}/F_{c_{u_3}} \sim m_c/m_t 1/\lambda^2$ , ...

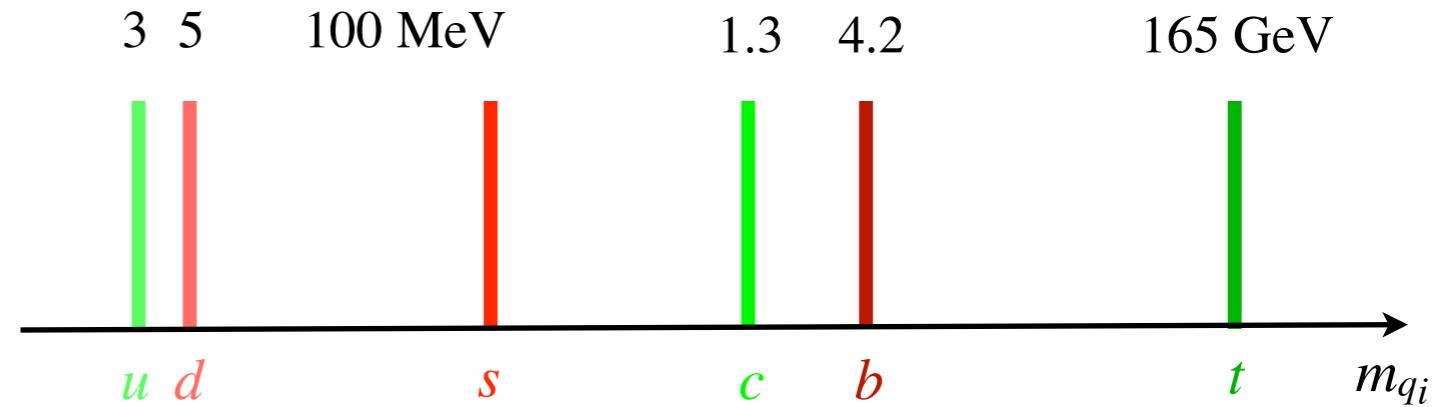
# Quark masses and mixings in RS model\*

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$$A = \mathcal{O}(1) \frac{F_{c_{Q_2}}^3}{F_{c_{Q_1}}^2 F_{c_{Q_3}}^2} ,$$

$$\bar{\rho} - i\bar{\eta} = \mathcal{O}(1)$$



$$V_{\text{CKM}} \approx \begin{pmatrix} 1 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda \approx 0.23, \quad A \approx 0.81, \quad \bar{\rho} \approx 0.14, \quad \bar{\eta} \approx 0.34$$

- To first order,  $\bar{\rho} - i\bar{\eta}$  is independent of fermion profiles  $F_{c_{Q_i}, c_{q_j}}$  at IR brane. Thus precise amount of CP violation remains unexplained in RS setup

# Physical parameters in quark sector\*

---

Flavor is violated by:

- |                                   |                                |
|-----------------------------------|--------------------------------|
| ▶ bulk parameters $c_Q, c_u, c_d$ | $3 \times 6$ real parameters   |
| - $3 \times 3$ hermitian matrices | $3 \times 3$ complex phases    |
| ▶ Yukawa couplings $Y_u, Y_d$     | $2 \times 9$ real parameters   |
| - $3 \times 3$ complex matrices   | $2 \times 9$ complex phases    |
| <hr/>                             |                                |
|                                   | 36 real parameters             |
|                                   | 27 complex phases              |
| ▶ global $U(3)^3$ flavor symmetry | 9 real parameters              |
| can be used to remove             | $18 - 1_B = 17$ complex phases |

Physical parameters:  $6_m + 12_\alpha + 9_c = 27$  moduli and  $1_{\text{CKM}} + 9_\phi = 10$  phases

# Warped-space Froggatt-Nielsen mechanism\*

Bulk fermions in RS:

$$(Y_q^{\text{eff,RS}})_{ij} \propto (Y_q)_{ij} e^{-kr\pi(c_{Q_i} - c_{q_j})}$$

Froggatt-Nielsen (FN) symmetry:

$$(Y_q^{\text{eff,FN}})_{ij} \propto (Y_q)_{ij} e^{(a_i - b_{q_j})}$$

- ▶ self-similarity along  $\phi$
  - ▶ bulk parameter  $c_{Q_i, q_i}$
  - ▶ IR brane at  $\phi = \pi$
  - ▶ warp factor  $e^{-2kr\pi}$
  - ▶  $U(1)_F$  symmetry
  - ▶  $U(1)_F$  charges  $Q_F = a_i, b_{q_i}$
  - ▶  $\langle \phi \rangle \neq 0$  of scalar  $\varphi$ ,  $Q_F = 1$
  - ▶  $\varepsilon = \langle h \rangle / \langle \phi \rangle \ll 1$
- 
- Models with warped spatial extra dimension provide compelling geometrical interpretation of flavor symmetry

# Reparametrization invariance\*

- Expressions for quark masses and mixing matrices are invariant under two reparametrizations RPI-1 and RPI-2

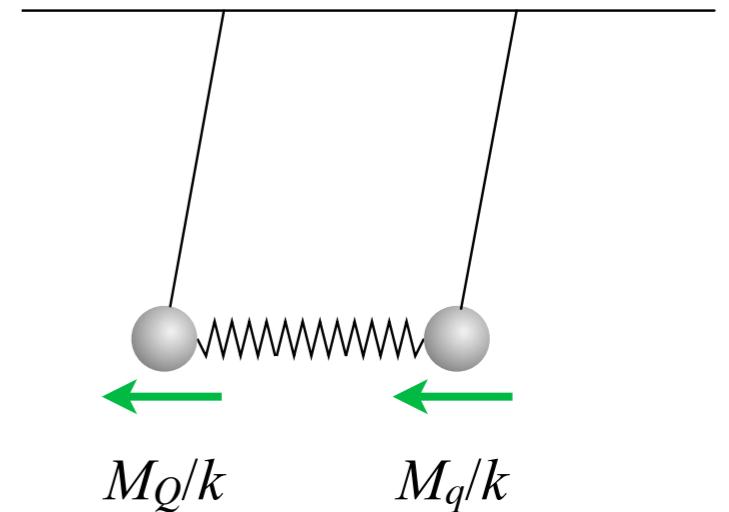
RPI-1:

$$F_{c_Q} \rightarrow e^{-\xi} F_{c_Q},$$

$$F_{c_q} \rightarrow e^{+\xi} F_{c_q},$$

$$\left[ c_Q \rightarrow c_Q - \frac{\xi}{L} \right],$$

$$\left[ c_q \rightarrow c_q + \frac{\xi}{L} \right]$$

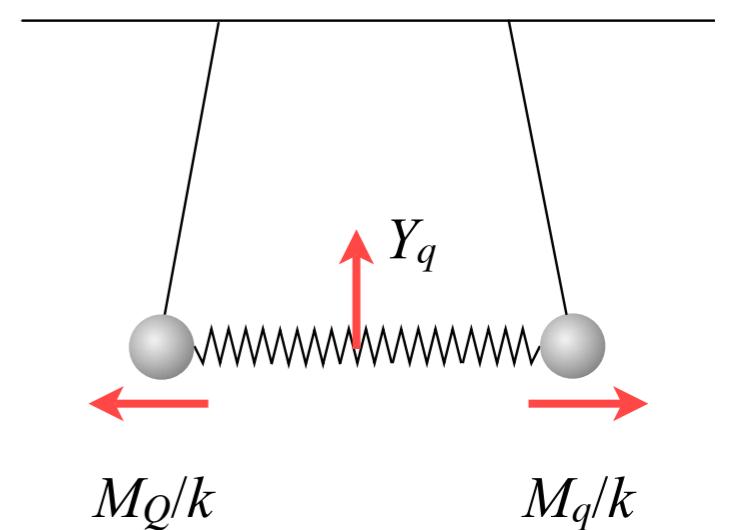


RPI-2:

$$F_{c_A} \rightarrow \zeta F_{c_A},$$

$$Y_q \rightarrow \frac{1}{\zeta^2} Y_q$$

$$\left[ c_A \rightarrow c_A - \frac{\ln \zeta}{L} \right],$$



# Mixing matrices: Transformation properties

---

RPI-1:

$$\begin{aligned}\Delta_Q &\rightarrow e^{-2\xi} \Delta_Q, & \Delta_q &\rightarrow e^{+2\xi} \Delta_q, \\ \delta_Q &\rightarrow e^{+2\xi} \delta_Q, & \delta_q &\rightarrow e^{-2\xi} \delta_q,\end{aligned}$$

RPI-2:

$$\begin{aligned}\Delta_Q &\rightarrow \zeta^2 \Delta_Q, & \Delta_q &\rightarrow \zeta^2 \Delta_q, \\ \delta_Q &\rightarrow \frac{1}{\zeta^2} \delta_Q, & \delta_q &\rightarrow \frac{1}{\zeta^2} \delta_q,\end{aligned}$$

Reparametrization transformations imply<sup>\*</sup>:

- ▶ relative importance of left- and right-handed couplings,  $\Delta_Q, \delta_Q \leftrightarrow \Delta_q, \delta_q$ , as well as contributions due to non-trivial gauge-boson profiles and fermion mixing,  $\Delta_{Q,q} \leftrightarrow \delta_{Q,q}$ , can be reshuffled
- ▶ but it is not possible to make all contributions simultaneously small

# Non-unitarity of CKM matrix\*

---

- Typical RS prediction:

$$1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = -0.00048,$$

$$1 + \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} + \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = -0.0068 + 0.0209 i$$

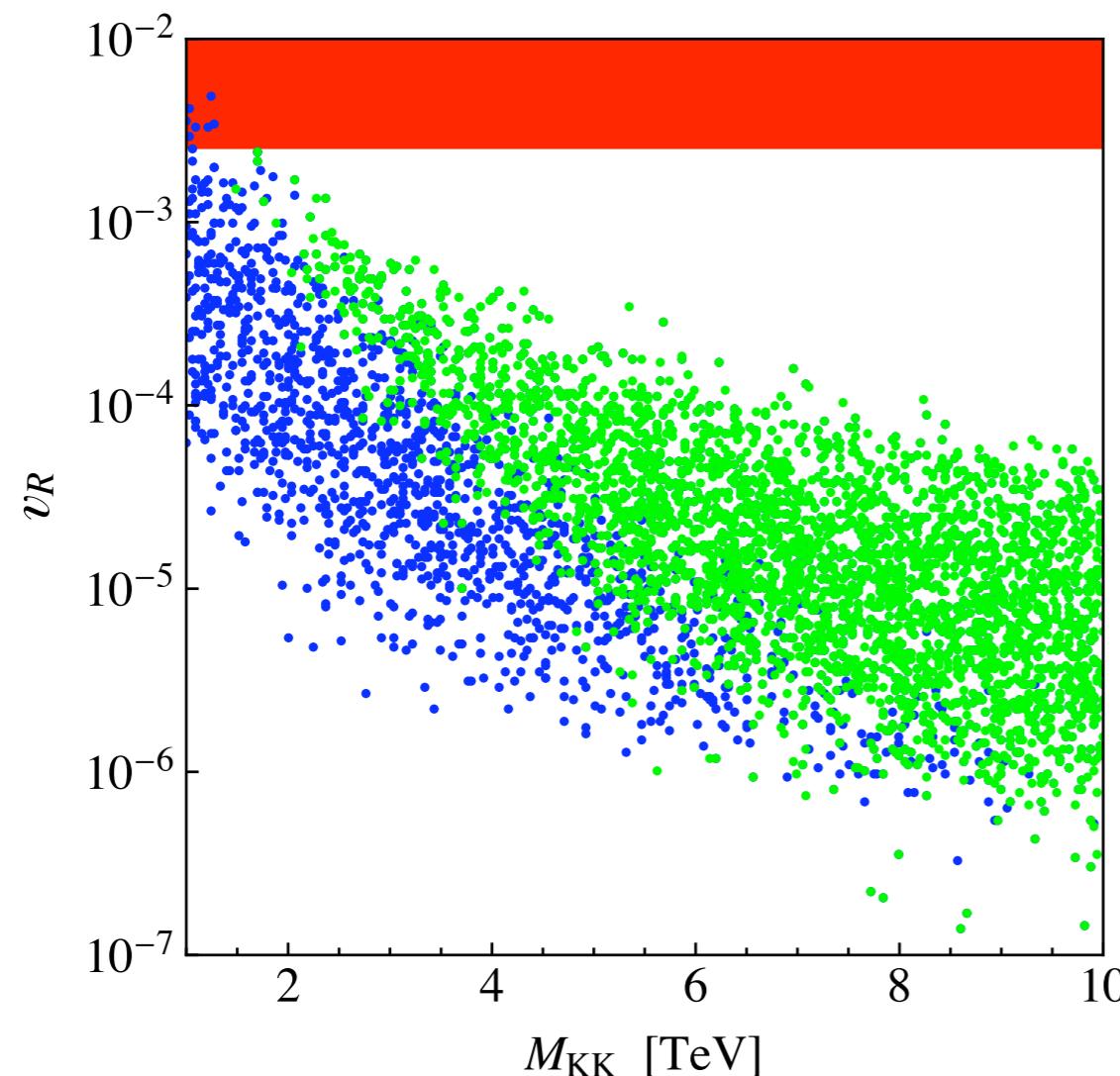
- Effects of similar magnitude as current uncertainties of global CKM fit:

$$1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = 0.00022 \pm 0.00051_{V_{ud}} \pm 0.00041_{V_{us}},$$

$$\bar{\rho} = 0.147 \pm 0.029, \quad \bar{\eta} = 0.343 \pm 0.016$$

# Right-handed charged current couplings\*

- Induced right-handed charged current couplings are too small to lead to observable effects. Most pronounced effects occur in  $Wtb$  coupling  $v_R$

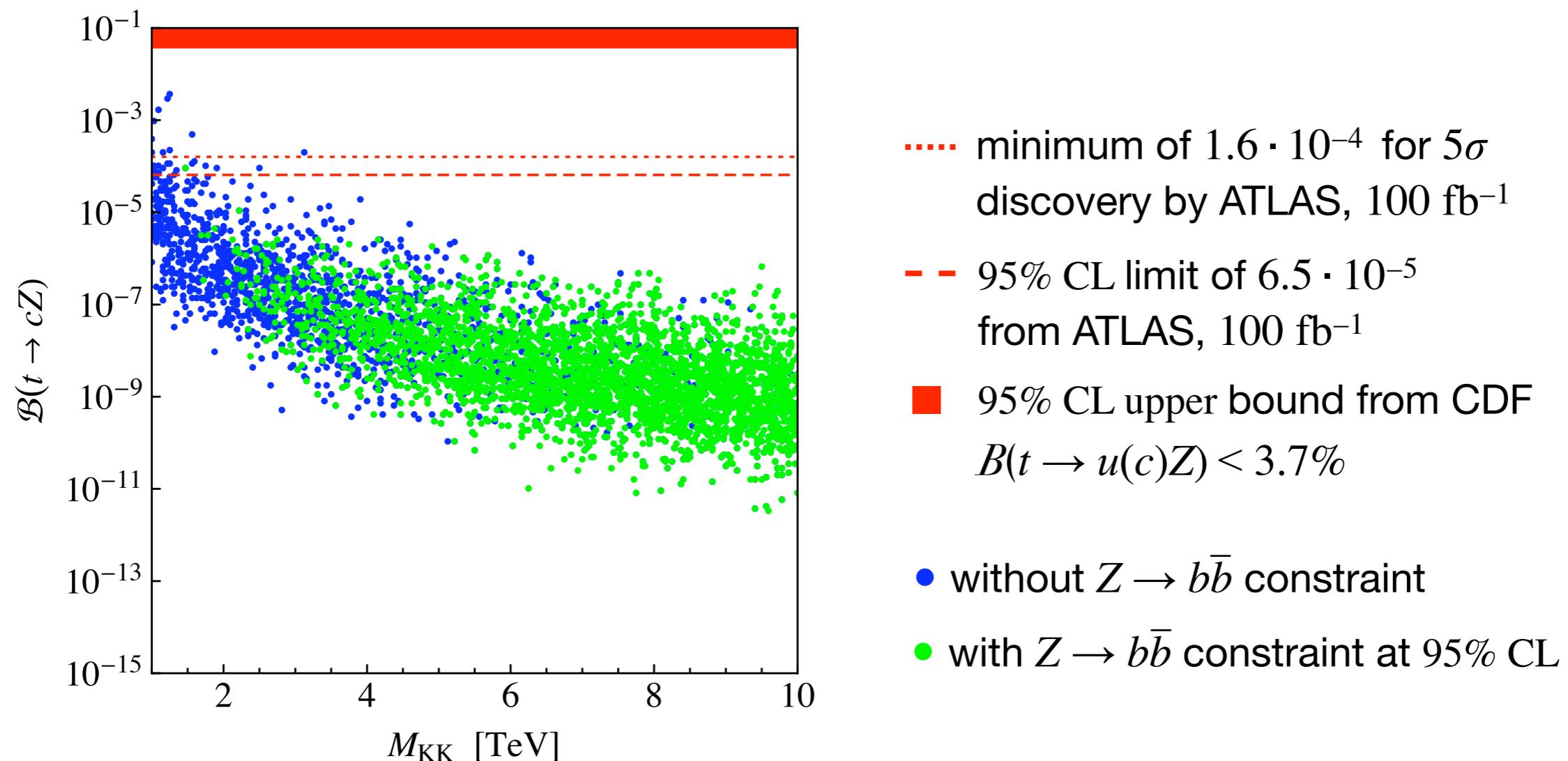


3000 randomly chosen RS points with  $|Y_q| < 3$  reproducing quark masses and CKM parameters with  $\chi^2/\text{dof} < 11.5/10$  corresponding to 68% CL

- $v_R \in [-0.0007, 0.0025]$  at 95% CL  
exclusion bound from  $B \rightarrow X_s \gamma$
- without  $Z \rightarrow b\bar{b}$  constraint
- with  $Z \rightarrow b\bar{b}$  constraint at 95% CL

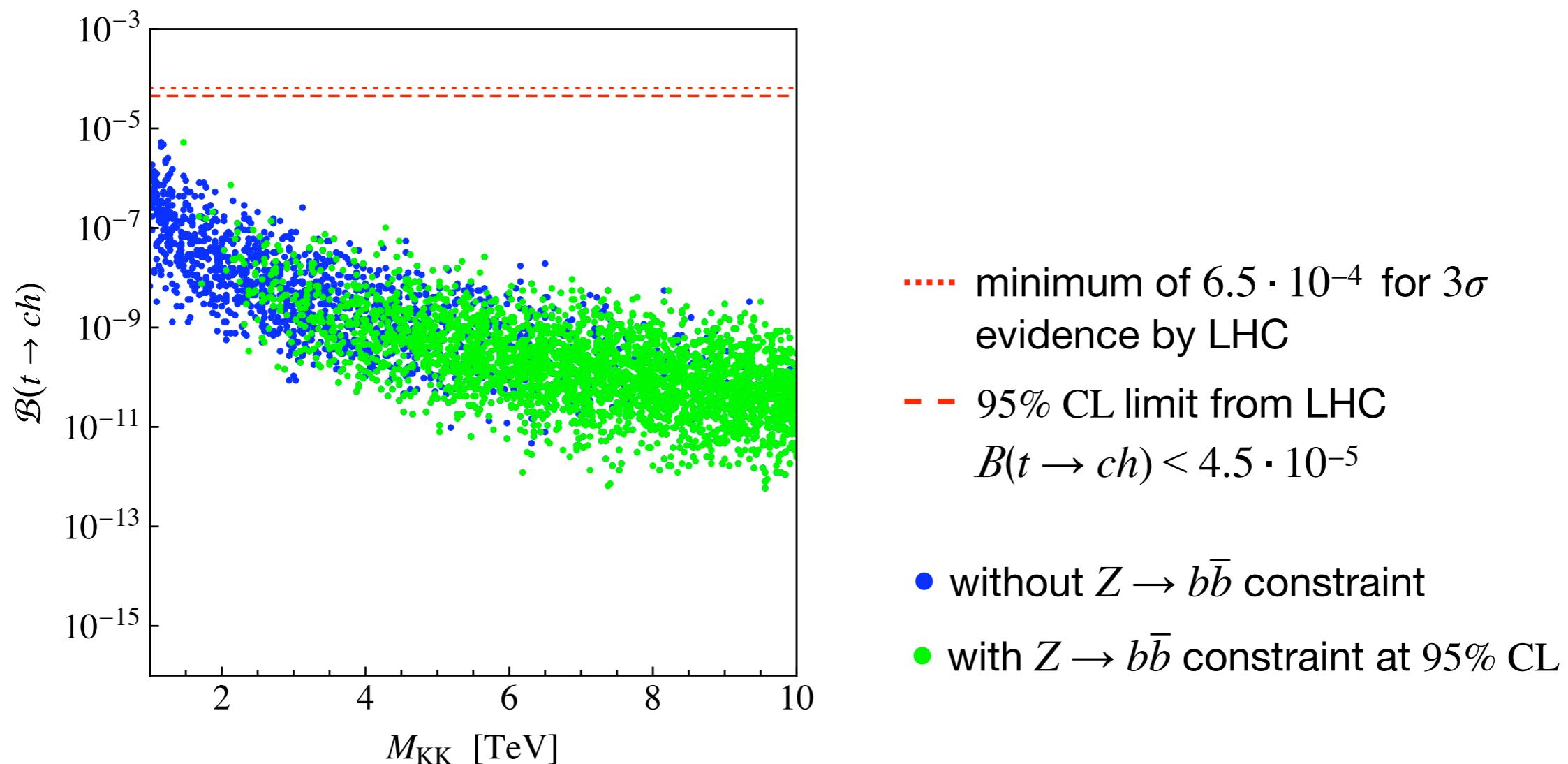
# Rare FCNC top decays\*

- Predictions of branching ratios for  $t \rightarrow cZ$  and  $t \rightarrow ch$  in minimal RS model typically below LHC sensitivity



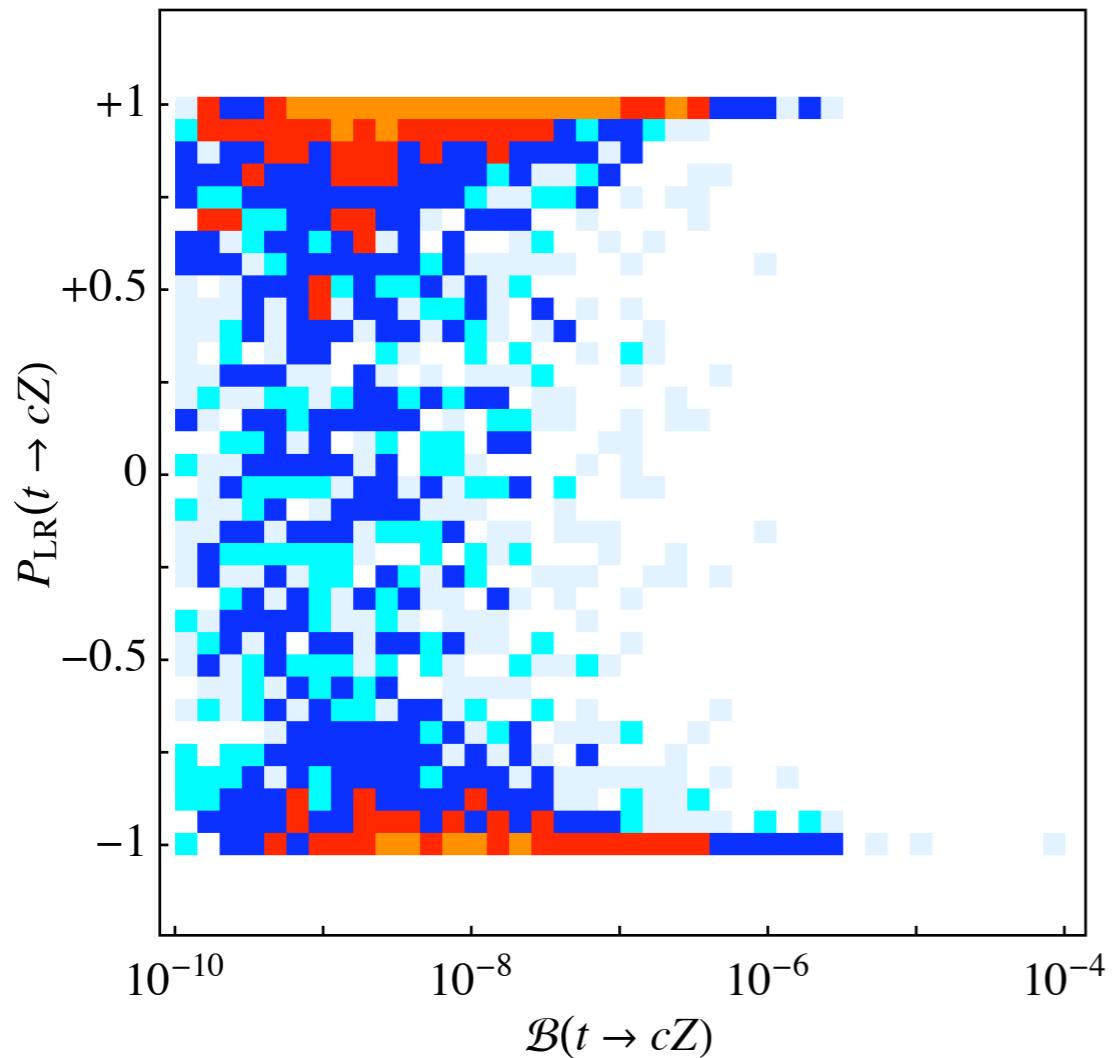
# Rare FCNC top decays\*

- Predictions of branching ratios for  $t \rightarrow cZ$  and  $t \rightarrow ch$  in minimal RS model typically below LHC sensitivity



# Rare FCNC top decays\*

- RS model does not lead to firm prediction for chirality of  $Ztc$  interactions, although  $Z \rightarrow b\bar{b}$  constraint restricts more strongly left-handed coupling



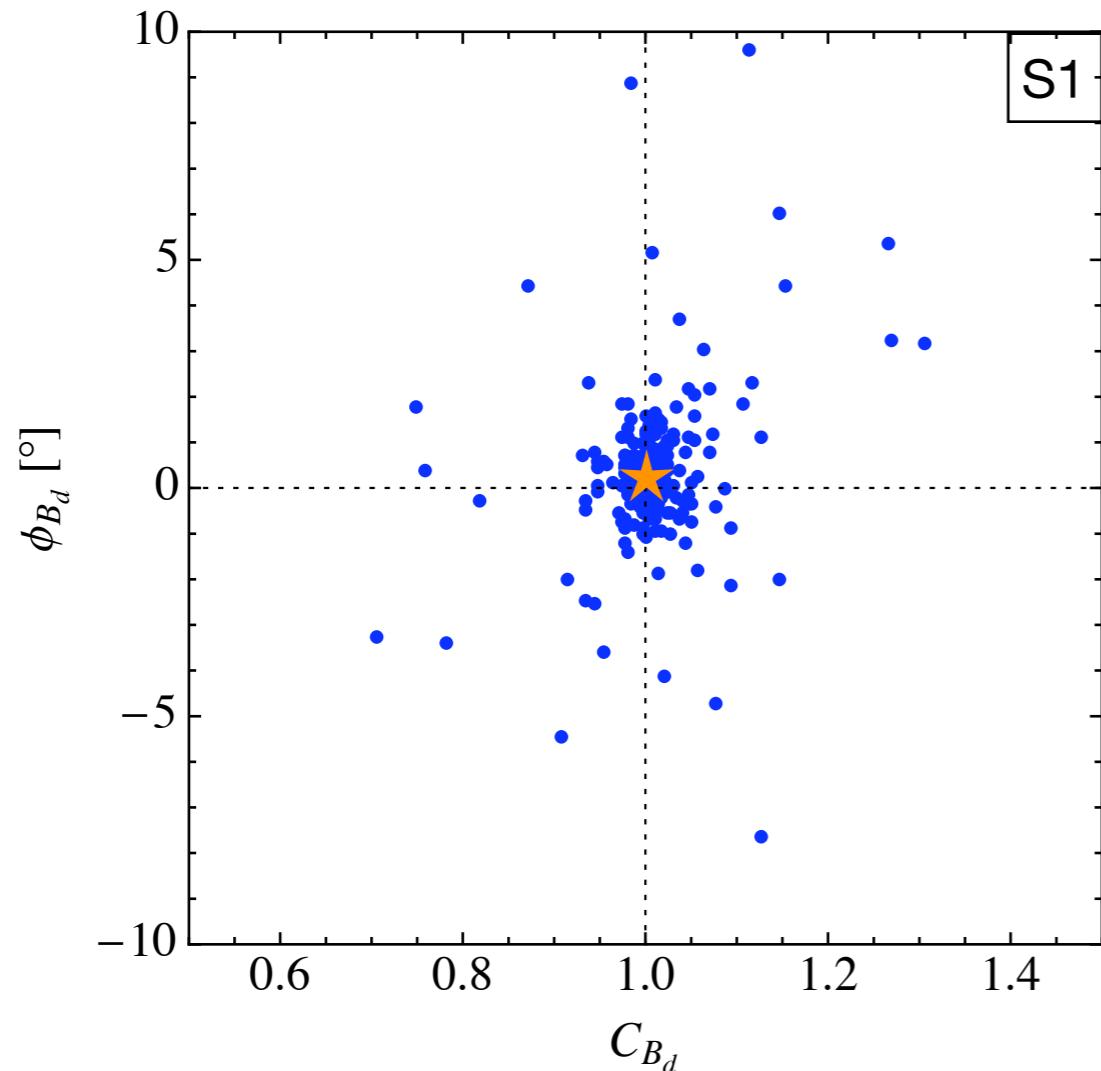
$$P_{LR}(t \rightarrow cZ) = \frac{\Gamma_L(t \rightarrow cZ) - \Gamma_R(t \rightarrow cZ)}{\Gamma_L(t \rightarrow cZ) + \Gamma_R(t \rightarrow cZ)}$$

■  $< 150$   
■  $< 50$   
■  $< 20$   
■  $< 7$   
■  $< 3$

events in bin after imposing  
 $Z \rightarrow b\bar{b}$  constraint at 95% CL

# Meson mixing: Neutral $B_d$ mesons\*

- Even after imposing  $|\varepsilon_K|$  constraint, sizable effects in magnitude and phase of  $B_d$  meson mixing amplitude possible



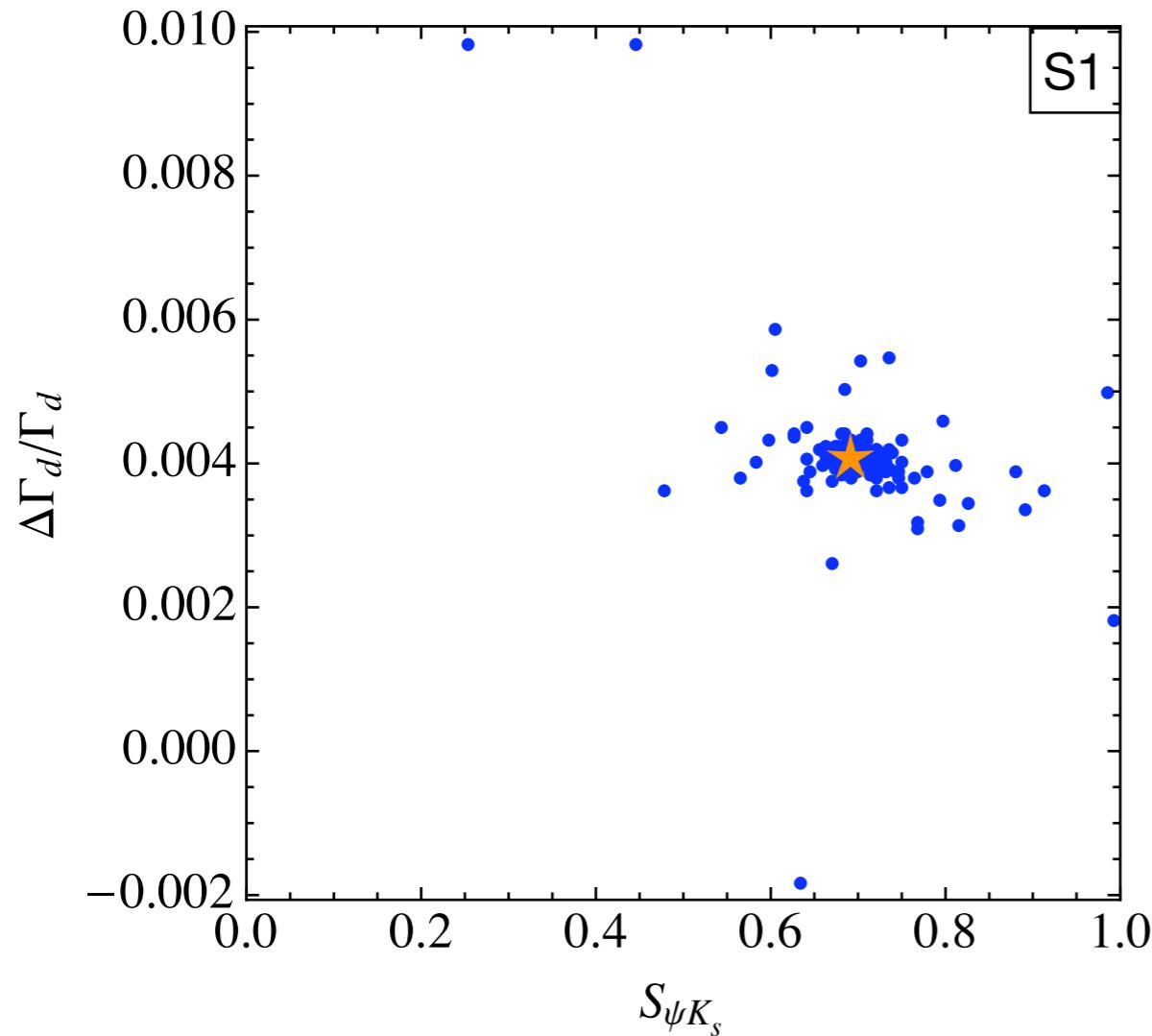
$$C_{B_d} e^{2i\phi_{B_d}} = \frac{\langle B_d | \mathcal{H}_{\text{eff},\text{full}}^{\Delta B=2} | \bar{B}_d \rangle}{\langle B_d | \mathcal{H}_{\text{eff},\text{SM}}^{\Delta B=2} | \bar{B}_d \rangle}$$

★ SM:  $C_{B_d} = 1, \phi_{B_d} = 0^\circ$

- consistent with quark masses, CKM parameters, and 95% CL limit  $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

# Meson mixing: Neutral $B_d$ mesons\*

- Constraint from  $|\varepsilon_K|$  does not exclude order one effects in width difference  $\Delta\Gamma_d/\Gamma_d$  of  $B_d$  system



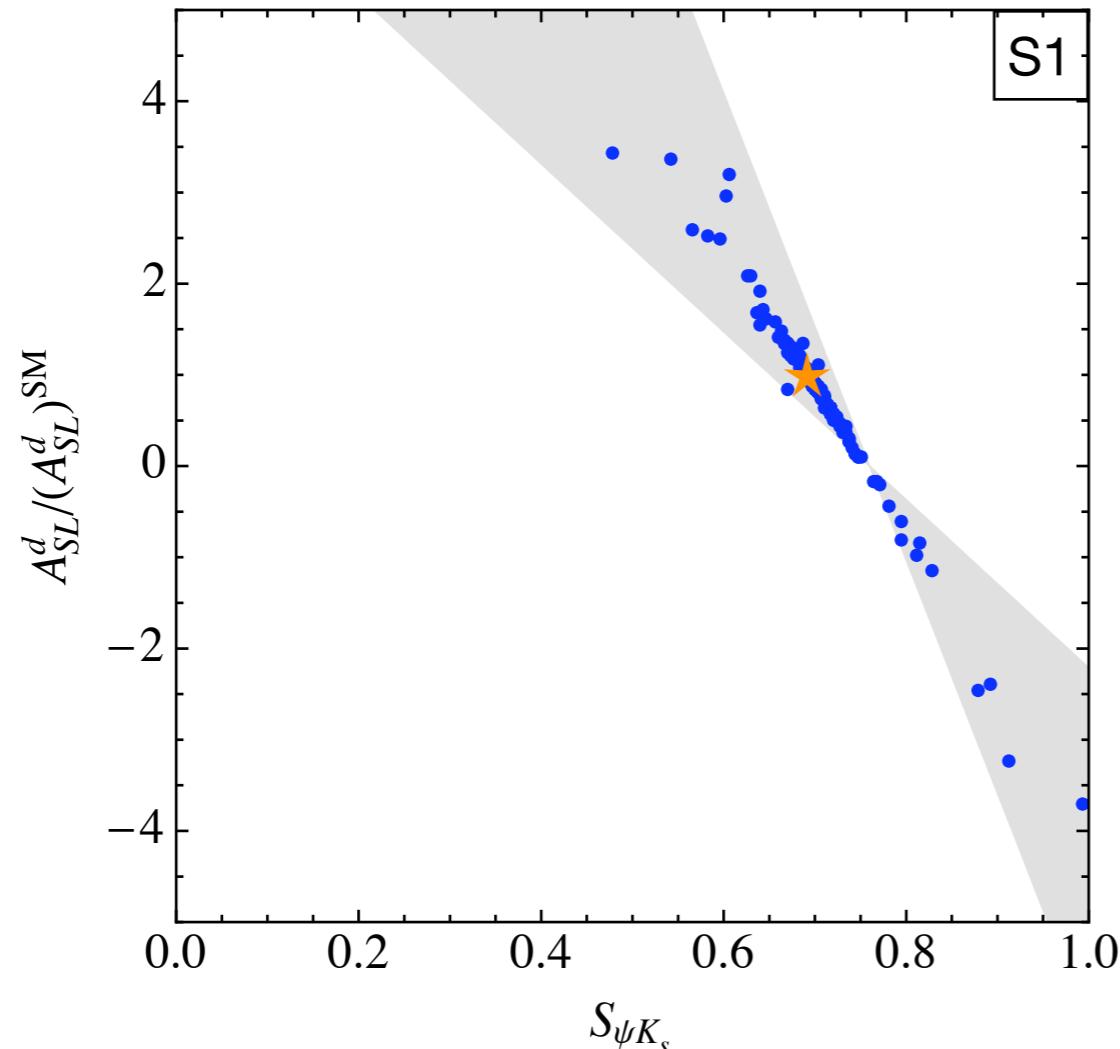
$$\begin{aligned}\Delta\Gamma_d &= \Gamma_L^d - \Gamma_S^d \\ &= 2 |\Gamma_{12}^d| \cos(2\beta + 2\phi_{B_d})\end{aligned}$$

★ SM:  $\Delta\Gamma_d/\Gamma_d \approx 0.004$ ,  $S_{\psi K_s} \approx 0.69$

- consistent with quark masses, CKM parameters, and 95% CL limit  $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

# Meson mixing: Neutral $B_d$ mesons\*

- In RS model, significant corrections to semileptonic CP asymmetry  $A_{SL}^d$  and  $S_{\psi K_S} = \sin(2\beta + 2\phi_{B_d})$  consistent with  $|\varepsilon_K|$  can arise



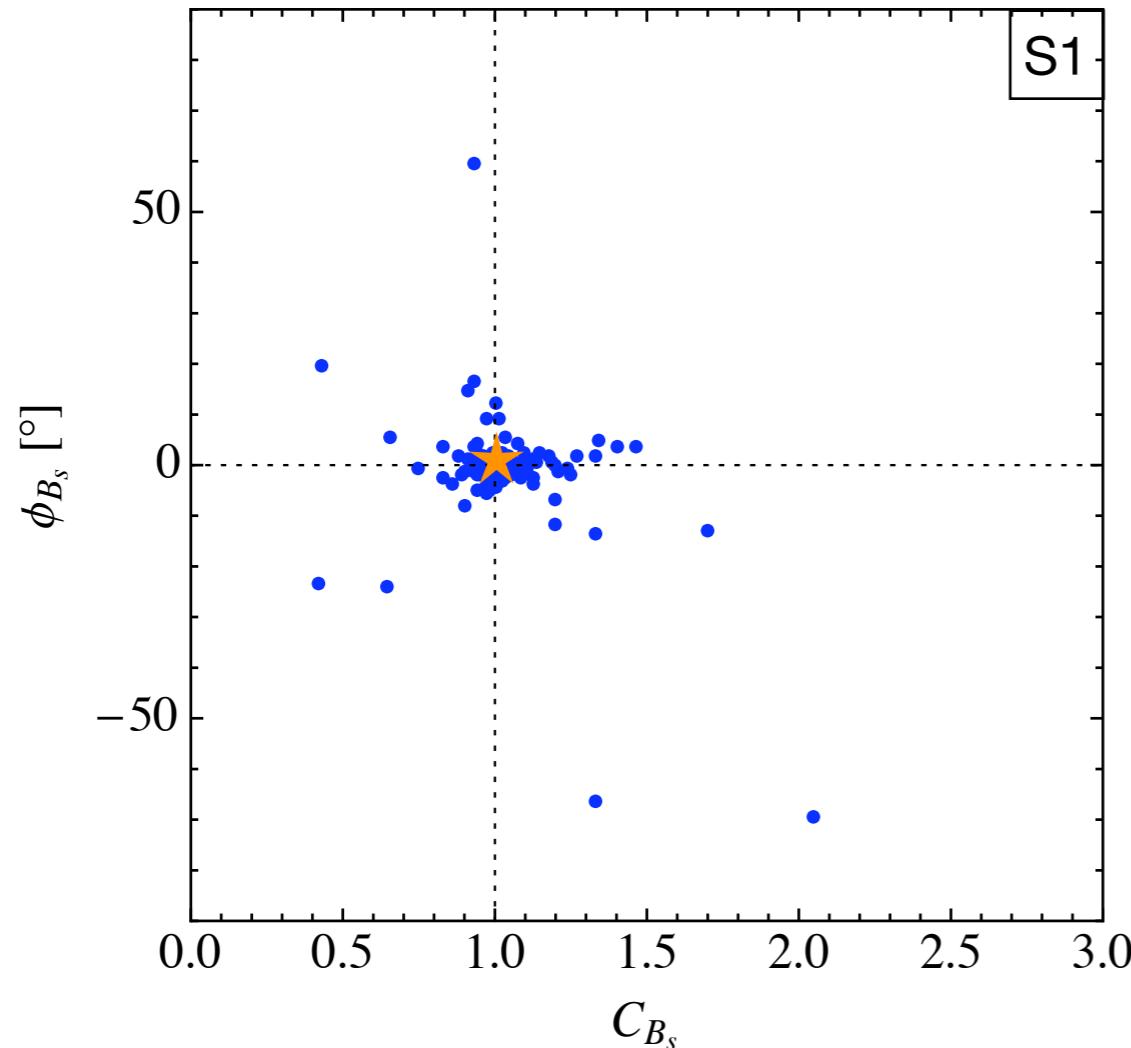
$$A_{SL}^d = \frac{\Gamma(\bar{B}_d \rightarrow l^+ X) - \Gamma(B_d \rightarrow l^- X)}{\Gamma(\bar{B}_d \rightarrow l^+ X) + \Gamma(B_d \rightarrow l^- X)}$$

$$= \text{Im} \left( \frac{\Gamma_{12}^d}{M_{12}^d} \right)$$

- ★ SM:  $A_{SL}^d \approx -5 \cdot 10^{-4}$ ,  $S_{\psi K_S} \approx 0.69$
- model-independent prediction
- consistent with quark masses, CKM parameters, and 95% CL  
limit  $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

# Meson mixing: Neutral $B_s$ mesons\*

- Even after imposing  $|\varepsilon_K|$  constraint, sizable effects in magnitude and phase of  $B_s$  meson mixing amplitude possible



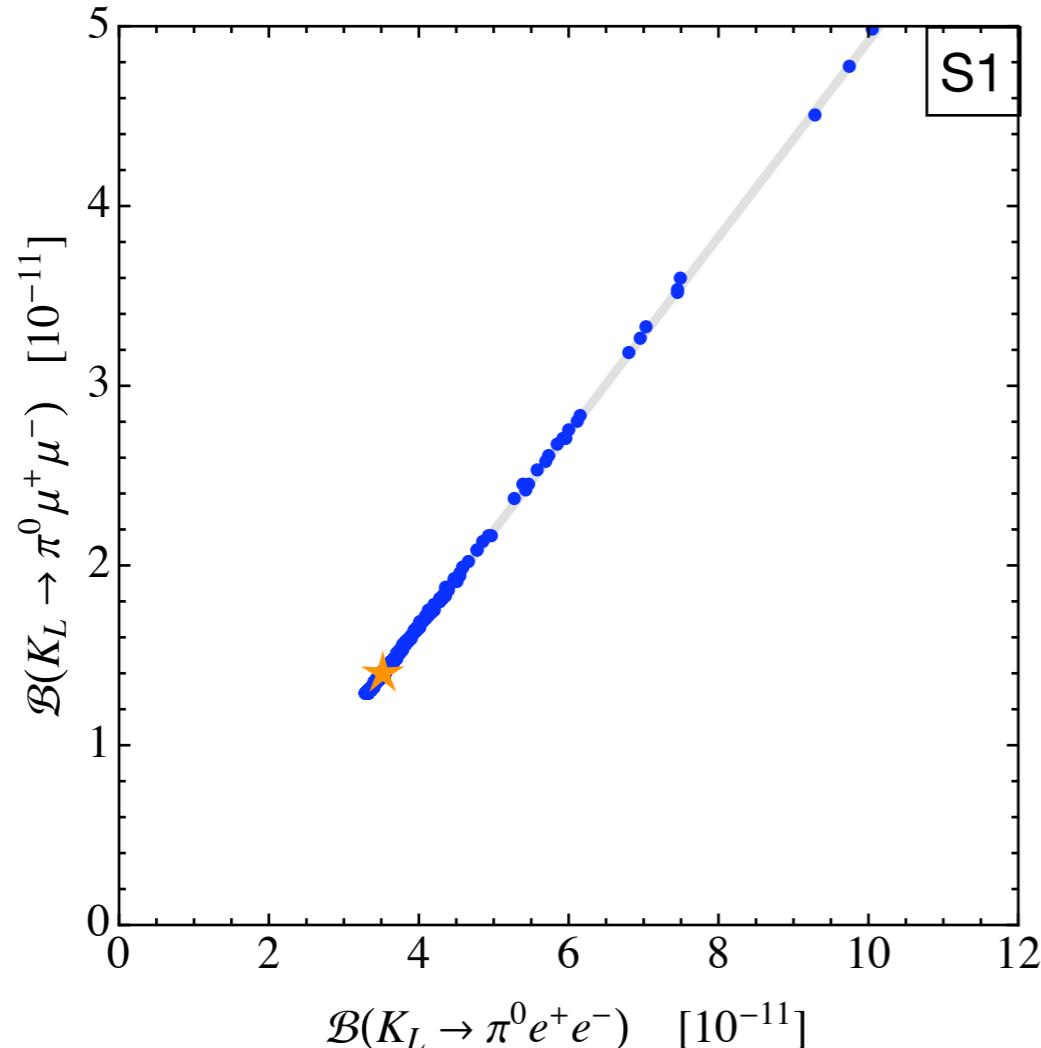
$$C_{B_s} e^{2i\phi_{B_s}} = \frac{\langle B_s | \mathcal{H}_{\text{eff},\text{full}}^{\Delta B=2} | \bar{B}_s \rangle}{\langle B_s | \mathcal{H}_{\text{eff},\text{SM}}^{\Delta B=2} | \bar{B}_s \rangle}$$

★ SM:  $C_{B_s} = 1, \phi_{B_s} = 0^\circ$

- consistent with quark masses, CKM parameters, and 95% CL limit  $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

# Rare $K$ decays: Silver modes\*

- Order one enhancements possible in  $K_L \rightarrow \pi^0 l^+ l^-$  modes. Effects in  $e^+ e^-$  and  $\mu^+ \mu^-$  channel are strongly correlated due to axial-vector dominance

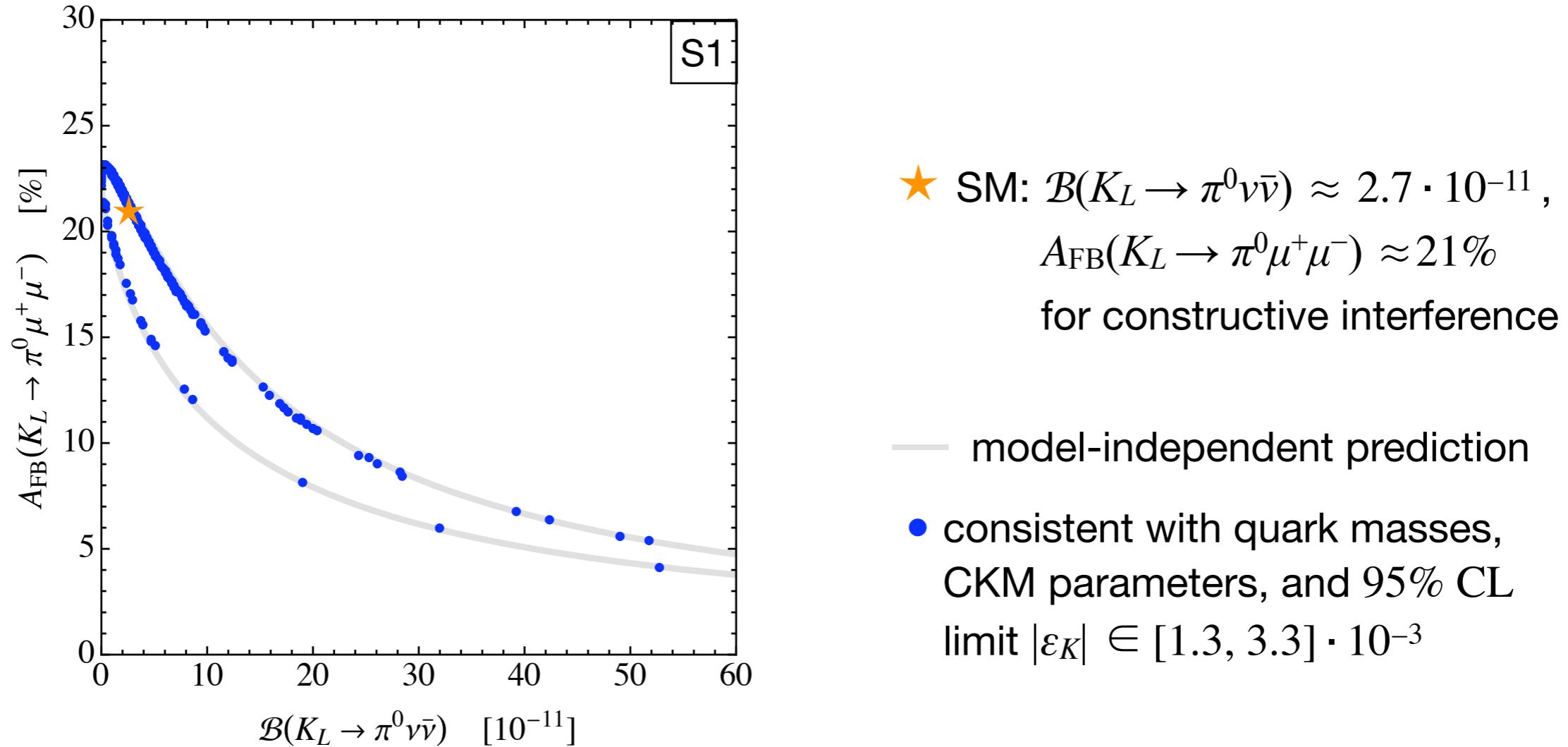


★ SM:  $\mathcal{B}(K_L \rightarrow \pi^0 e^+ e^-) \approx 3.6 \cdot 10^{-11}$ ,  
 $\mathcal{B}(K_L \rightarrow \pi^0 \mu^+ \mu^-) \approx 1.4 \cdot 10^{-11}$   
for constructive interference

- model-independent prediction
- consistent with quark masses, CKM parameters, and 95% CL limit  $|\varepsilon_K| \in [1.3, 3.3] \cdot 10^{-3}$

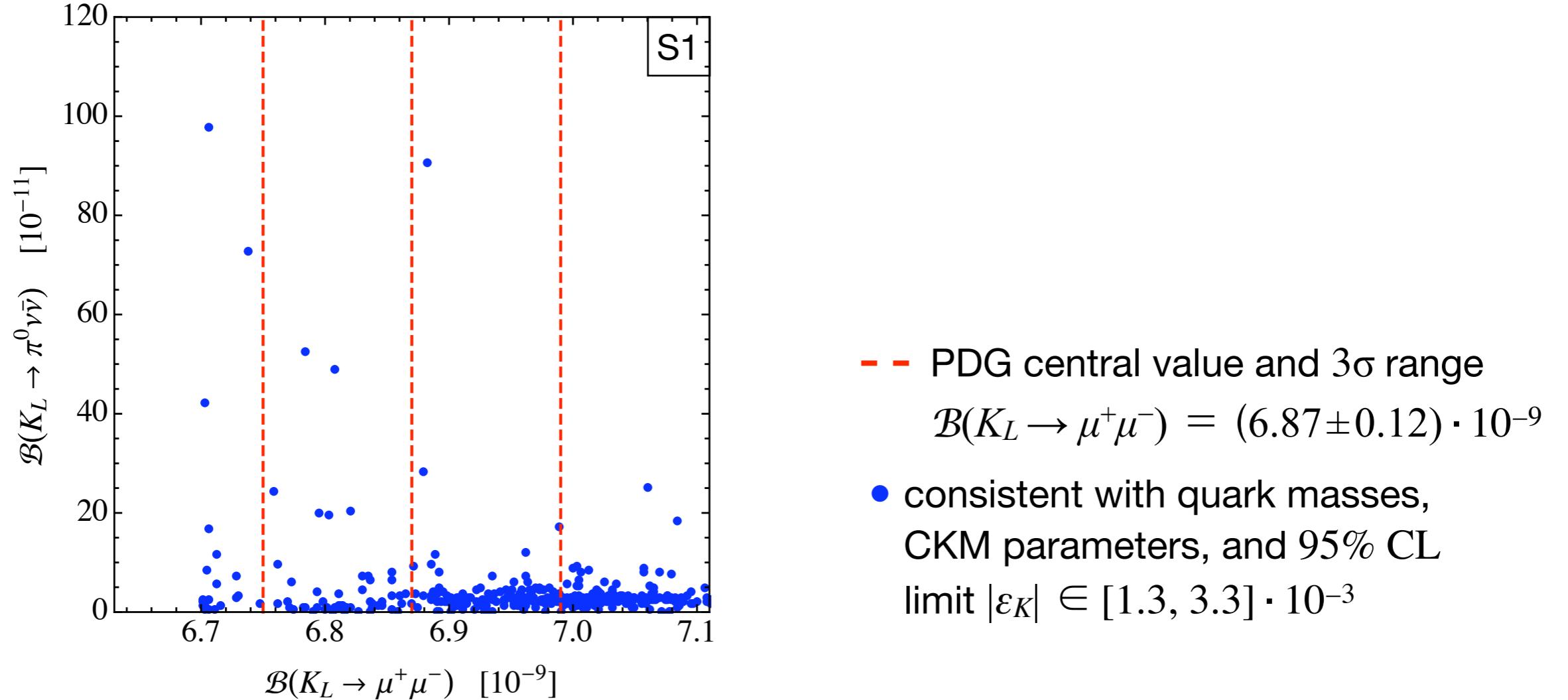
# Rare $K$ decays: Silver modes\*

- Deviations from SM expectations in  $K_L \rightarrow \pi^0\nu\bar{\nu}$  and  $K_L \rightarrow \pi^0l^+l^-$  follow specific pattern, arising from smallness of vector and scalar contributions



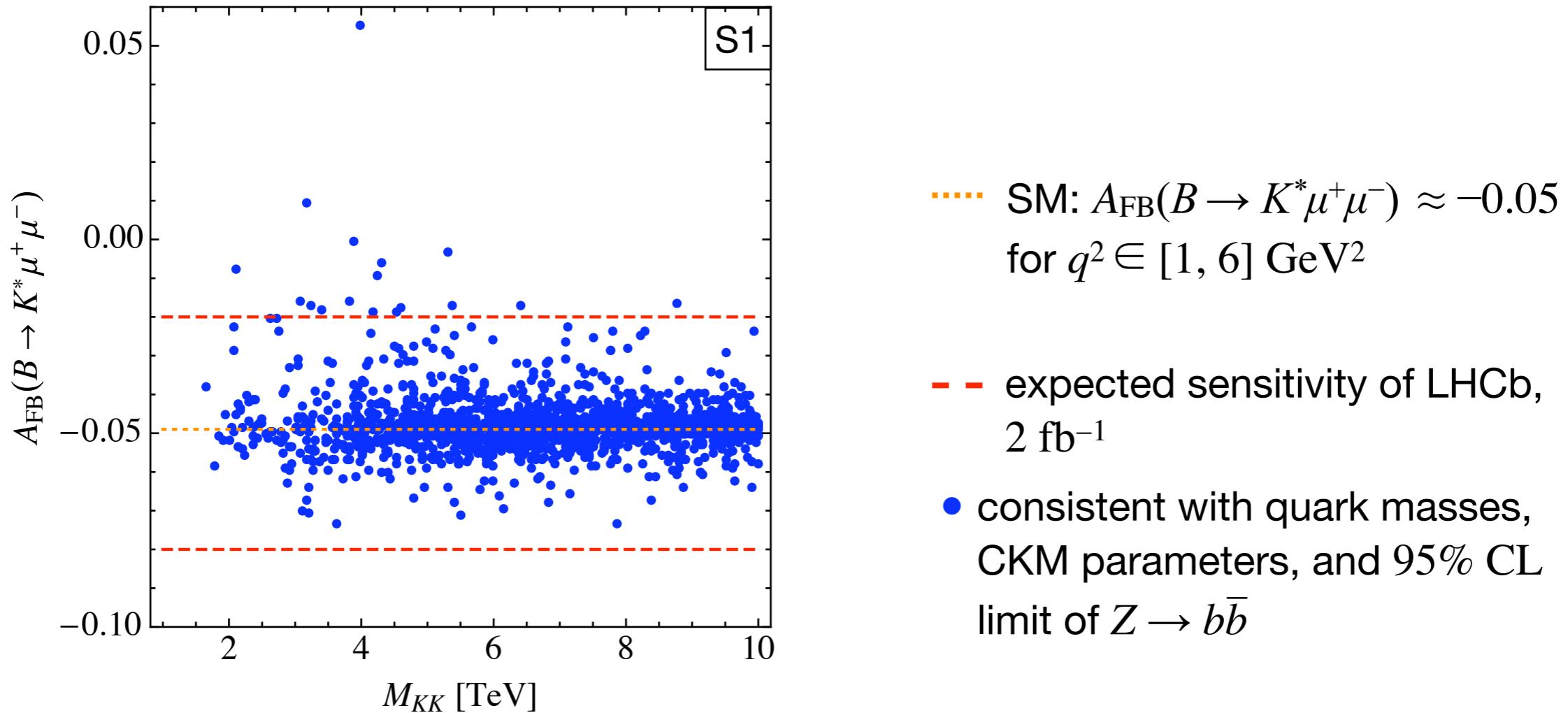
# Rare $K$ decays: Bronze mode\*

- Better theoretical understanding of precisely measured  $K_L \rightarrow \mu^+ \mu^-$  mode could allow to constrain possible enhancement of  $K_L \rightarrow \pi^0 \nu \bar{\nu}$



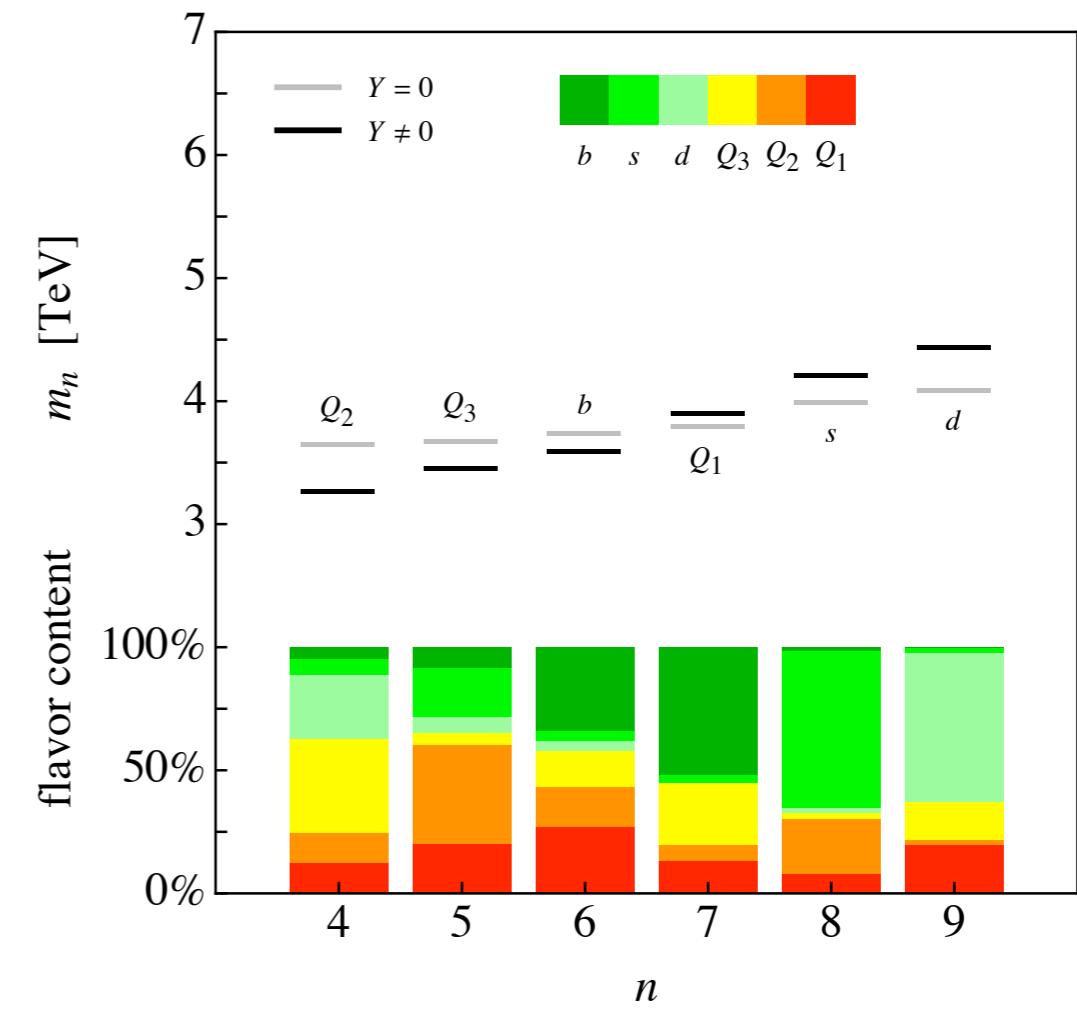
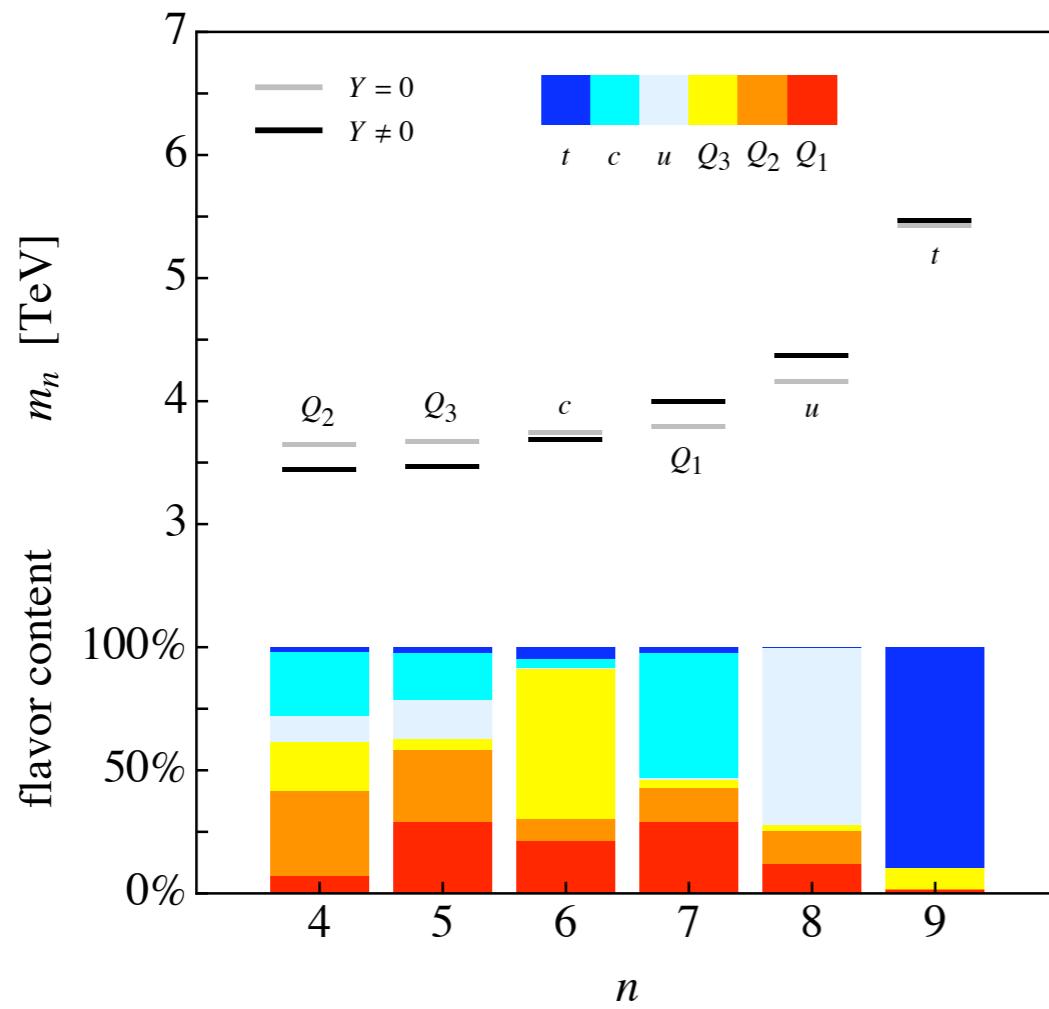
# Rare $B$ decays: Exclusive semileptonic modes\*

- Corrections to  $A_{\text{FB}}(B \rightarrow K^* \mu^+ \mu^-)$  on average below LHCb sensitivity. Other angular distributions such as  $A_T^{(3)}(B \rightarrow K^* \mu^+ \mu^-)$  might offer better prospects



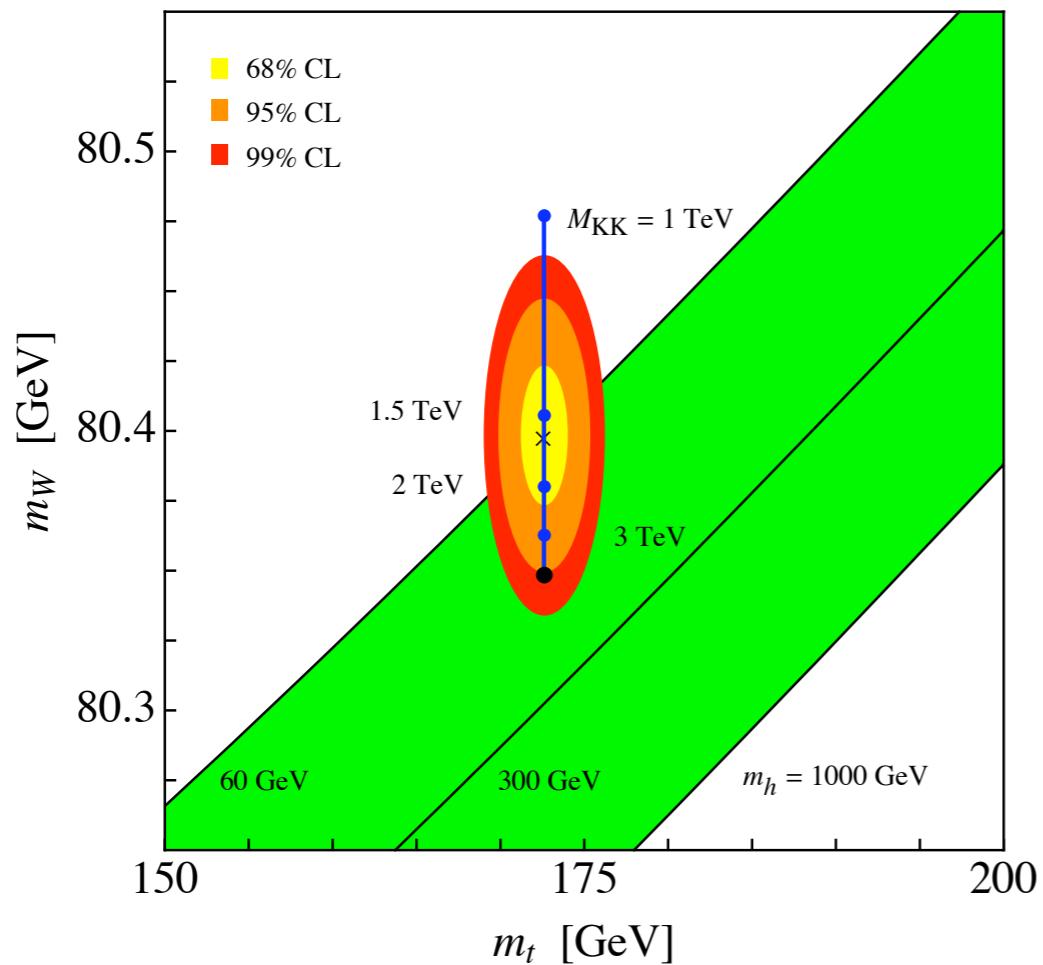
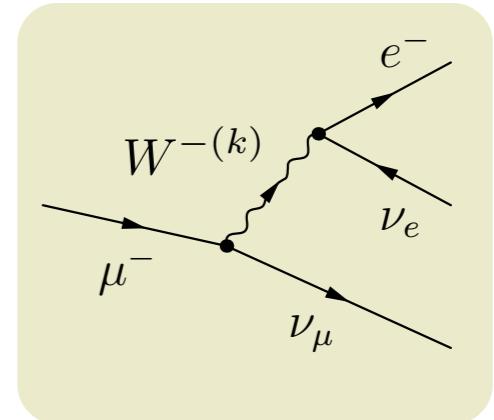
# Mass and mixing of KK fermions\*

- Since mass splittings of undisturbed KK states typical of order 100 GeV order, Yukawa couplings introduce large mixings among KK modes of same level. Mixings give rise to FCNCs when inserted into loop diagrams



# Mass of $W$ boson\*

- RS model allows to explain 50 MeV difference between direct and indirect extractions of  $W$ -boson mass  $m_W \approx 80.40$  GeV and  $(m_W)_{\text{ind}} \approx 80.35$  GeV

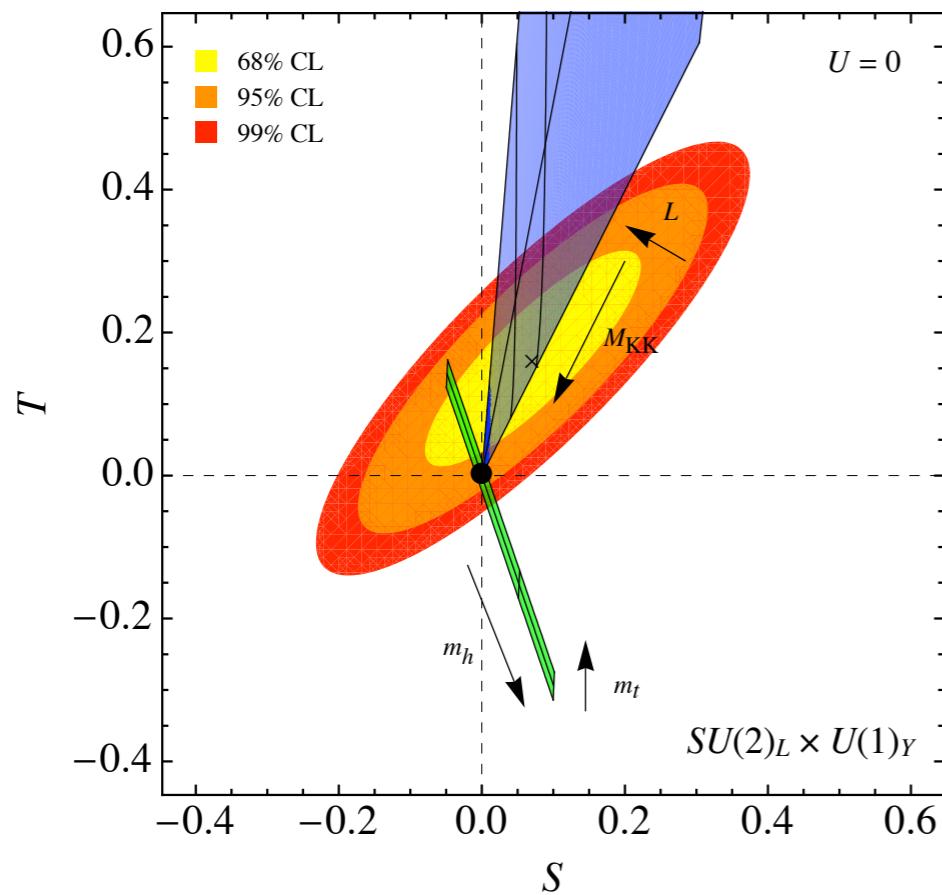


$$(m_W)_{\text{ind}} \approx m_W \left[ 1 - \frac{m_W^2}{4M_{\text{KK}}^2} \left( 1 - \frac{1}{2L} \right) \right]$$

- $(m_W)_{\text{ind}}$  in SM for  $m_h \in [60, 1000]$  GeV
- $(m_W)_{\text{ind}}$  in SM for  $m_h = 150$  GeV
- $(m_W)_{\text{ind}}$  in RS model for  $M_{\text{KK}} \in [1, 3]$  TeV

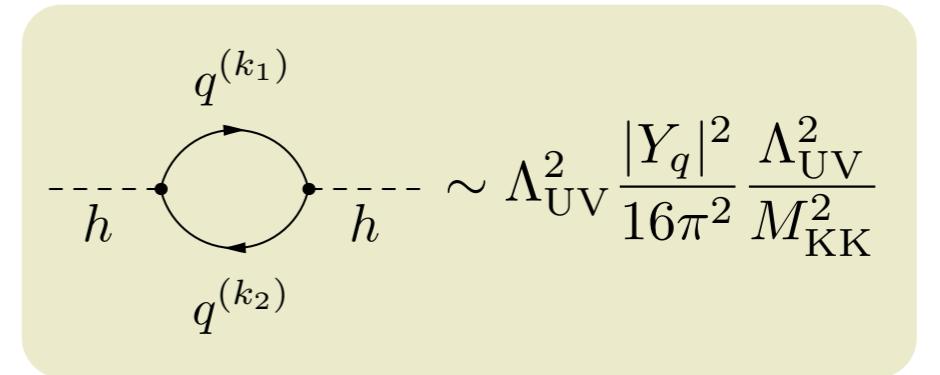
# $S$ and $T$ parameters in minimal RS model\*

- In warped models with brane-localized Higgs sector,  $m_h$  naturally of order  $M_{\text{KK}}$ . Heavy Higgs allows for  $M_{\text{KK}} > 2.6 \text{ TeV}$  at 99% CL consistent with  $S$  and  $T$



$$\Delta S = \frac{1}{6\pi} \ln \frac{m_h}{m_h^{\text{ref}}} , \quad \Delta T = -\frac{3}{8\pi c_w^2} \ln \frac{m_h}{m_h^{\text{ref}}}$$

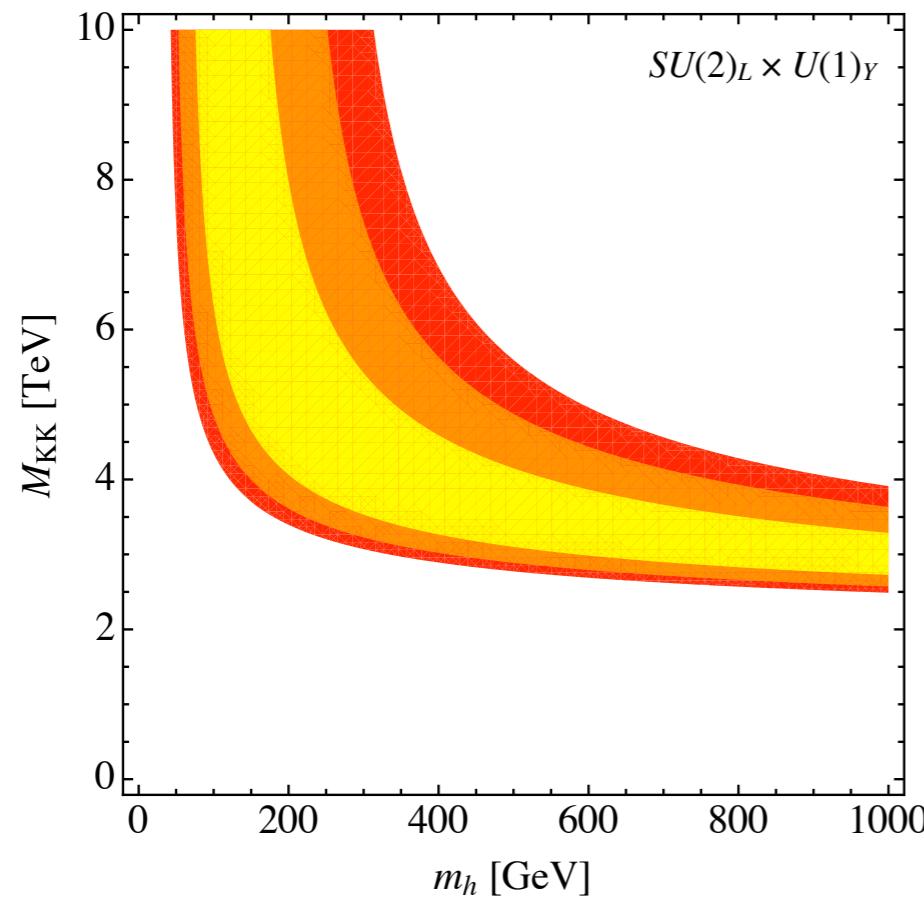
- minimal RS prediction for  $M_{\text{KK}} \in [1, 10] \text{ TeV}$  and  $L \in [5, 37]$
- SM reference point for  $m_h \in [60, 1000] \text{ GeV}$  and  $m_t = (172.6 \pm 1.4) \text{ GeV}$
- SM reference point for  $m_h = 150 \text{ GeV}$



# $S$ and $T$ parameters in minimal RS model\*

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$$\sim \Lambda_{\text{UV}}^2 \frac{|Y_q|^2}{16\pi^2} \frac{\Lambda_{\text{UV}}^2}{M_{\text{KK}}^2}$$



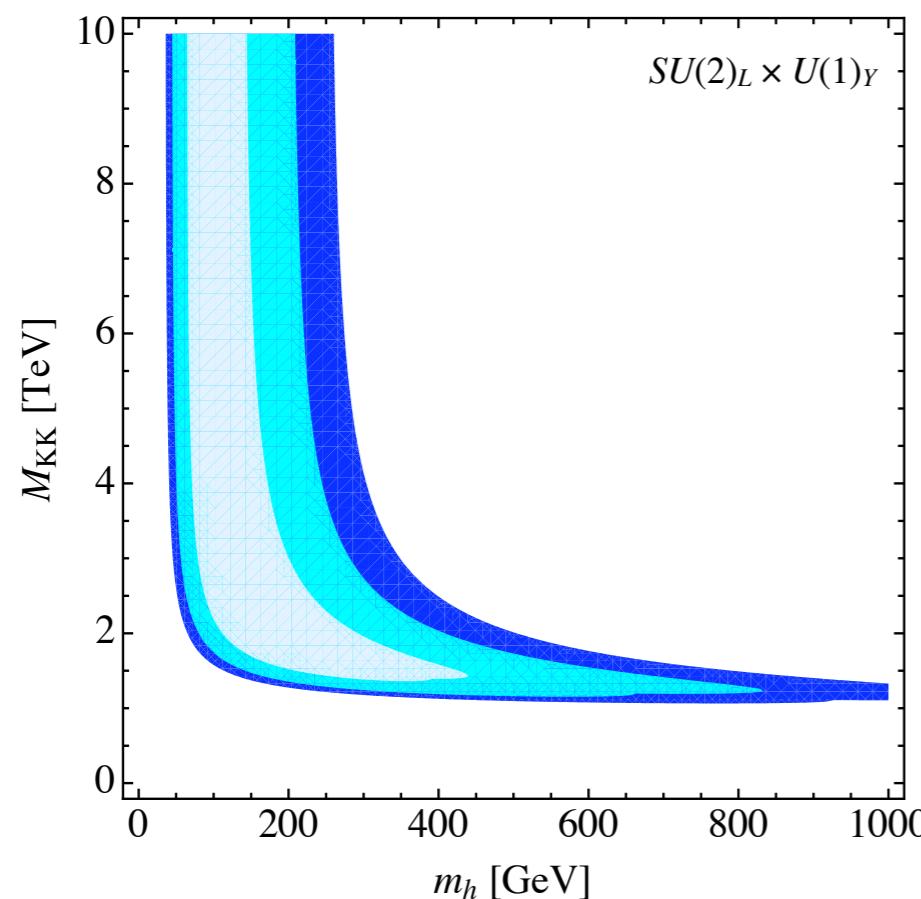
$$\Delta S = \frac{1}{6\pi} \ln \frac{m_h}{m_h^{\text{ref}}} , \quad \Delta T = -\frac{3}{8\pi c_w^2} \ln \frac{m_h}{m_h^{\text{ref}}}$$

■ 68% CL  
■ 95% CL  
■ 99% CL

regions from  $S$  and  $T$  in minimal RS model for  $L = \ln(10^{16}) \approx 37$

# $S$ and $T$ parameters in little RS model\*

- Another way to protect  $T$  from vast corrections consists in giving up on solution to full gauge hierarchy problem by working in volume-truncated RS background. For  $L = \ln(10^3) \approx 7$ , allowed KK scale is lowered to  $M_{\text{KK}} > 1.5$  TeV at 99% CL for  $m_h = 150$  GeV

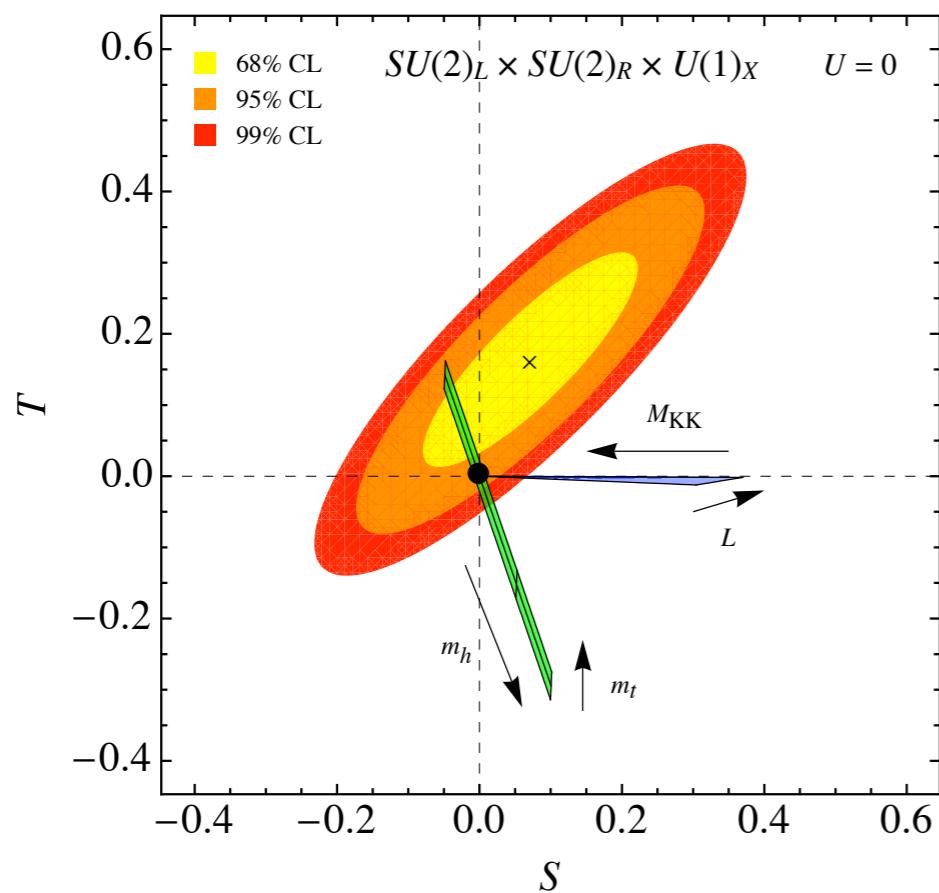


$$S = \frac{2\pi v^2}{M_{\text{KK}}^2} \left( 1 - \frac{1}{L} \right) ,$$
$$T = \frac{\pi v^2}{2c_w^2 M_{\text{KK}}^2} \left( L - \frac{1}{2L} \right)$$

■ 68% CL  
■ 95% CL      regions from  $S$  and  $T$  in little RS  
■ 99% CL      model for  $L = \ln(10^3) \approx 7$

# $S$ and $T$ parameters in extended RS model\*

- Most elegant cure for excessive contributions to  $T$  parameter is custodial  $SU(2)_R$  symmetry. Lower bound of KK scale follows then from constraint on  $S$ . For  $m_h = 150$  GeV one finds  $M_{\text{KK}} > 2.4$  TeV at 99% CL. Yet presence of heavy Higgs boson could spoil global electroweak fit

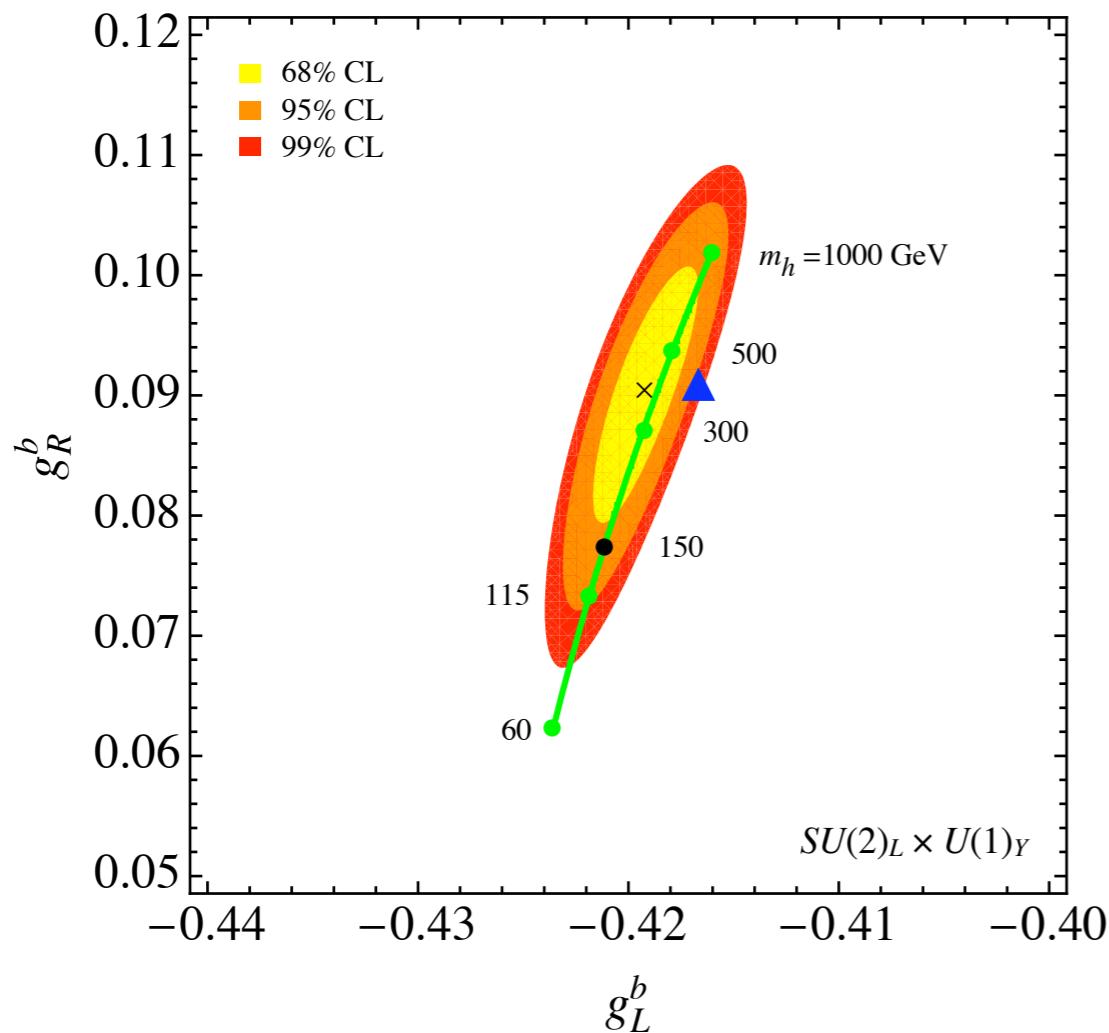


$$S = \frac{2\pi v^2}{M_{\text{KK}}^2} \left(1 - \frac{1}{L}\right), \quad T = -\frac{\pi v^2}{4c_w^2 M_{\text{KK}}^2} \frac{1}{L}$$

- prediction in extended RS model for  $M_{\text{KK}} \in [1, 10]$  TeV and  $L \in [5, 37]$
- SM reference point for  $m_h \in [60, 1000]$  GeV and  $m_t = (172.6 \pm 1.4)$  GeV
- SM reference point for  $m_h = 150$  GeV

# $Z \rightarrow b\bar{b}$ in minimal RS model\*

- Heavy Higgs boson improves quality of fit to pseudo observables  $R_b^0$ ,  $A_b$ , and  $A_{FB}^{0,b}$ . Minimal RS model thus offer indirect explanation of  $2.1\sigma$  anomaly in  $A_{FB}^{0,b}$  since in this setup Higgs-boson mass is expected to large



$$\Delta A_{FB}^{0,b} = -2.7 \cdot 10^{-3} \ln \frac{m_h}{m_h^{\text{ref}}}$$

- minimal RS prediction for reference point with  $M_{KK} = 1.5$  TeV and  $m_h = 400$  GeV
- SM prediction for  $m_h \in [60, 1000]$  GeV
- SM prediction for  $m_h = 150$  GeV